Hecht, Henryk; Miličić, Dragan

On the cohomological dimension of the localization functor. (English) Zbl 0714.22011

Proc. Am. Math. Soc. 108, No. 1, 249-254 (1990).

The localization functor of the title is defined by $\Delta_\lambda(V) = \mathcal{D}_\lambda \otimes_{\mathcal{U}_\lambda} V$, where λ is an element of the dual of a fixed Cartan subalgebra of a complex semisimple Lie algebra $\mathfrak{g}$, $\mathcal{U}_\lambda$ the quotient of the enveloping algebra of $\mathfrak{g}$ by the ideal generated by the maximal ideal of its centre corresponding in the appropriate way to the orbit of λ under the (shifted) action of the Weyl group, V a $\mathcal{U}_\lambda$-module, and $\mathcal{D}_\lambda$ the sheaf of twisted differential operators corresponding to λ on the flag variety of $\mathfrak{g}$ as defined by A. Beilinson and J. Bernstein. The main theorem says that for λ singular the left cohomological dimension of Δ_λ is infinite, which contrasts with a result proved by *A. Beilinson* and *J. Bernstein* for λ regular [Representation theory of reductive groups, Prog. Math. 40, 35-52 (1983; Zbl 0526.22013)].

Reviewer: H.de Vries

MSC:

22E47 Representations of Lie and real algebraic groups: algebraic methods (Verma modules, etc.)

22E46 Semisimple Lie groups and their representations

Keywords:

localization functor; Cartan subalgebra; complex semisimple Lie algebra; enveloping algebra; action; Weyl group; sheaf of twisted differential operators; flag variety; left cohomological dimension

Full Text: DOI

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