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Characteristic classes for algebraic vector bundles with Hermitian metric. I. (English)

Zbl 0715.14018

Ann. Math. (2) 131, No. 1, 163-203 (1990).

The authors define characteristic classes for arbitrary hermitian vector bundles on arithmetic varieties (extending Arakelov theory). Let X be a regular, flat, quasiprojective variety over \mathbb{Z} . Let E be an algebraic vector bundle on X and h an hermitian metric on the corresponding holomorphic vector bundle E_∞ on the set X_∞ of complex points of X . One defines a Chern character $\text{ch}(E, h)$ in the graded group $CH(X)_\mathbb{Q} = \bigoplus_{p \geq 0} CH^p(X) \otimes \mathbb{Q}$ (where $CH^p(X)$ is the arithmetic Chow group of codimension p of X), which is characterized by the properties: functoriality, additivity, multiplicativity, normalization and compatibility with Chern forms. One proves that ch induces an isomorphism from $K_0(X) \otimes_{\mathbb{Z}} \mathbb{Q}$ to $CH(X)_\mathbb{Q}$.

This part I of this paper has four sections: In section 1, one studies Bott-Chern secondary characteristic classes and in section 2, one studies the first Chern class of an hermitian line bundle in the language of this paper. In section 3, a splitting principle for CH of Grassmannians is proved. Using this principle, in section 4, one defines characteristic classes and one proves their properties.

[For part II of this paper see *ibid.*, No.2, 205-238 (1990; Zbl 0715.14006)].

Reviewer: Vasile Brînzănescu

MSC:

14G40 Arithmetic varieties and schemes; Arakelov theory; heights
14F05 Sheaves, derived categories of sheaves, etc. (MSC2010)
57R10 Smoothing in differential topology

Cited in **6** Reviews
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Keywords:

characteristic classes for arbitrary hermitian vector bundles on arithmetic varieties; Arakelov theory; Chern class

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