Rota, Rosaria
Strongly distributive multiplicative hyperrings. (English) Zbl 0716.16023
J. Geom. 39, No. 1-2, 130-138 (1990).

A hyperring is a triple $(A,+,\circ)$ where $(A,+)$ is an abelian group, $a\circ b$ is a subset of A for each a,b$\in A$, and the following axioms are satisfied $\forall a,b,c\in A:$ (i) $a\circ(b\circ c)=(a\circ b)\circ c$ (ii) $(a+b)\circ c\subseteq a\circ c+b\circ c$ (iii) $a\circ(b+c)\subseteq a\circ b+a\circ c$ (iv) $(-a)\circ b=a\circ(-b)=-(a\circ b)$. If equality holds in (ii) (resp. (iii)) then A is called strongly left (right) distributive. If A is both strongly left and strongly right distributive, then it is called a strongly distributive hyperring. Various results are proved for strongly left and right hyperrings, in particular the following: Let $(A,+,\circ)$ be a strongly left (right) hyperring such that, for any a,b$\in A|a\circ b|=k>1$. Then $(A,+,\circ)$ is also strongly right (left) distributive.

The author then gives attention to strongly distributive hyperrings, and obtains the following result inter alia: Let $(A,+,\circ)$ be a strongly distributive hyperring. If a ring $(R,+,.)$ exists, together with a bijection $\alpha:R\to A$ such that $\forall x,y\in R$, (i) $\alpha(x+y)=\alpha(x)+\alpha(y)$; (ii) α (x$\cdot y)\in\alpha(x)\circ\alpha(y)$, then if we denote by S the hyperideal $0\circ 0$ and $T=\alpha^{-1}(S)$, T is an ideal of R and the quotient $(R,+,.)/T$ is isomorphic to the ring $(A,+,\circ)/S$. Moreover, it is possible to define in $(A,+)$ a product $\times$ in order to obtain a ring $(A,+,\times)$ isomorphic to $(R,+,.)$ through α. Furthermore, $(A,+,\times)/S$ will be isomorphic to $(R,+,.)/T$ and $\forall a,b\in A$, $a\circ b=a\times b+S$. A similar result is obtained under a somewhat different hypothesis.

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