Goodwillie, Thomas G.
A multiple disjunction lemma for smooth concordance embeddings. (English) ⟨Zbl 0717.57011⟩
Mem. Am. Math. Soc. 431, 317 p. (1990).

An element of C(P,N) is an embedding of $I \times P$ in $I \times N$ agreeing with the identity on $O \times P \cup I \times \partial P$. The object of the paper is to show that if P, $Q_1, ..., Q_r$ are disjoint submanifolds of N of respective codimensions a, $b_1, ..., b_r$ then the homotopy groups of the $(r+1)-ad$ $(C(P,N); C(P, N-Q_1), ..., C(P, N-Q_r))$ vanish in dimensions $\leq a - 3 + \sum_1^r (b_i - 1)$. This is part of an overall strategy to determine the homotopy type of C(P,N). The proof proceeds by constructing a stratification of $P \times C(P,N)$, essentially to take into account the singularities of the composite of the evaluation $I \times P \times C(P,N) \to I \times N$ with projection on N: in fact the strata are defined by restrictions on multijets $I \times P \to I \times N$. The definition is inductive: the initial step measures the deviation of the concordance from being an isotopy; then four operations are defined; roughly speaking, these take into account non-smooth points, double points of the map, and singular points of the map. The construction of the stratification and verification of its basic properties occupies 120 pp. Part of the proof consists in inductively constructing homotopies of a map from a disc to C(P,N), to avoid successively more strata. The prototype homotopy has the effect of pushing the singularity off the top edge of the concordance, and the author is motivated by the technique of 'sunny collapsing'. Thus for multiple points of a projection to N the order of heights in I is important, and plays a key role in showing that the argument fails for certain 'bad' strata.

Although the author is at pains to give a lengthy introduction explaining the strategy of the proof, the presentation in detail is very difficult to follow.

Reviewer: C.T.C.Wall

MSC:

57Q60	Cobordism and concordance in PL-topology
58D10	Spaces of embeddings and immersions
57R45	Singularities of differentiable mappings in differential topology

Cited in **7** Documents

Keywords:

homotopy groups of n-ads; concordance embeddings; isotopy; non-smooth points; double points; singular points

Full Text: DOI