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Isospectral Hamiltonian flows in finite and infinite dimensions. II: Integration of flows.
(English) [Zbl 0717.58051](#)
Commun. Math. Phys. 134, No. 3, 555-585 (1990).

In Part I of this paper [ibid. 117, No.3, 451-500 (1988; [Zbl 0659.58022](#))], the authors showed how isospectral Hamiltonian flows in the space of rank r perturbations, \mathcal{M}_A , of an $n \times n$ matrix A can be derived from the Adler-Kostant-Symes theorem. These flows arise through the use of a moment map from \mathcal{M}_A into the dual, $(\mathfrak{gl}(r)^+)^*$, of the positive part of the loop algebra $\mathfrak{gl}(r)$. Such systems were shown to be completely integrable under special assumptions on the spectrum of A and the resulting matrix polynomial $L(\lambda) \in (\mathfrak{gl}(r)^+)^*$.

The purpose of this part II is to provide a more unified, streamlined formulation which allows A and $L(\lambda)$ to have more general spectra. Such a generalization is necessary to be able to treat important examples of integrable systems such as the coupled non-linear Schrödinger equation (CNLS). The authors illustrate their general constructions by explicitly solving CNLS as well as the Rosochatius equation.

Reviewer: [W.J.Satzer jun](#)

MSC:

- [37C10](#) Dynamics induced by flows and semiflows
- [37J35](#) Completely integrable finite-dimensional Hamiltonian systems, integration methods, integrability tests
- [37K10](#) Completely integrable infinite-dimensional Hamiltonian and Lagrangian systems, integration methods, integrability tests, integrable hierarchies (KdV, KP, Toda, etc.)
- [35Q55](#) NLS equations (nonlinear Schrödinger equations)

Cited in **2** Reviews
Cited in **36** Documents

Keywords:

isospectral flows; complete integrability; coupled non-linear Schrödinger equation

Full Text: [DOI](#)

References:

- [1] Adams, M. R., Harnad, J., Previato, E.: Isopectral Hamiltonian flows in finite and infinite dimensions, I. Generalized Moser systems and moment maps into loop algebras. *Commun. Math. Phys.* 117, 451–500 (1988) · [Zbl 0659.58022](#) · [doi:10.1007/BF01223376](#)
- [2] Adams, M. R., Harnad, J., Hurtubise, J.: Liouville generating functions for isospectral Hamiltonian flows in loop algebras. *Integrable and Superintegrable Systems*. Kupershmidt, B. (ed.) Singapore: World Scientific (in press 1990); Darboux coordinates and Liouville-Arnold integration in loop algebras. (In preparation) · [Zbl 0791.58047](#)
- [3] Adler, M., van Moerbeke, P.: Completely integrable systems, euclidean Lie algebras, and curves. *Adv. Math.* 38, 267–317 (1980); Linearization of Hamiltonian systems, Jacobi varieties, and representation theory. *Adv. Math.* 38, 318–379 (1980) · [Zbl 0455.58017](#) · [doi:10.1016/0001-8708\(80\)90007-9](#)
- [4] Dubrovin, B. A.: Theta Functions and non-linear equations. *Russ. Math. Surv.* 36, 11–92 (1981) · [Zbl 0549.58038](#) · [doi:10.1070/RM1981v036n02ABE](#)
- [5] Dubrovin, B. A., Matveev, V. B., Novikov, S. P.: Non-linear equations of Korteweg de Vries type, finite zone linear operators, and abelian varieties. *Russ. Math. Surv.* 31, 59–146 (1976) · [Zbl 0346.35025](#) · [doi:10.1070/RM1976v031n01ABEH001446](#)
- [6] Griffiths, P., Harris, J.: Principles of algebraic geometry. New York: John Wiley 1978 · [Zbl 0408.14001](#)
- [7] Gagnon, L., Harnad, J., Hurtubise, J., Winternitz, P.: Abelian integrals and the reduction method for an integrable Hamiltonian system. *J. Math. Phys.* 26, 1605–1612 (1985) · [Zbl 0597.70020](#) · [doi:10.1063/1.526926](#)
- [8] Guillemin, V., Sternberg, S.: The moment map and collective motion. *Ann. Phys.* 127, 220–253 (1980) · [Zbl 0453.58015](#) · [doi:10.1016/0003-4916\(80\)90155-4](#)
- [9] Hitchin, N.: On the construction of monopoles. *Commun. Math. Phys.* 89, 145–190 (1983) · [Zbl 0517.58014](#) · [doi:10.1007/BF01211826](#)
- [10] Hurtubise, J.: Rankr perturbations, algebraic curves and ruled surfaces, preprint
- [11] Krichever, I. M., Novikov, S. P.: Holomorphic bundles over algebraic curves and non-linear equations. *Russ. Math. Surv.* 35, 6, 53–79 (1980) · [Zbl 0548.35100](#) · [doi:10.1070/RM1980v035n06ABEH001974](#)

- [12] Krichever, I. M.: Algebraic curves and commuting matrix differential operators. *Funct. Anal. Appl.*10, 144–146 (1976); *Methods of algebraic geometry in the theory of non-linear equations*. *Russ. Math. Surv.*32, 6, 198–213 (1977) · [Zbl 0347.35077](#) · [doi:10.1007/BF01077946](#)
- [13] Moser, J.: *Geometry of quadrics and spectral theory*. The Chern symposium, Berkeley, June 1979; pp. 147–188. Berlin, Heidelberg, New York: Springer 1980
- [14] Mumford, D.: *Tata lectures on theta*. II. *Prog. Math.* 43. Boston: Birkhäuser 1983 · [Zbl 0509.14049](#)
- [15] van Moerbeke, P., Mumford, D.: The spectrum of difference operators and algebraic curves. *Acta Math.*143, 93–154 (1979) · [Zbl 0502.58032](#) · [doi:10.1007/BF02392090](#)
- [16] Previato, E.: Hyperelliptic quasi-periodic and soliton solutions of the nonlinear Schrödinger equation. *Duke Math. J.*52 · [Zbl 0578.35086](#)
- [17] Reyman, A. G., Semenov-Tian-Shansky, M. A.: Reduction of Hamiltonian systems, affine Lie algebras and Lax equations II. *Invent. Math.*63, 423–432 (1981) · [Zbl 0452.58014](#) · [doi:10.1007/BF01389063](#)
- [18] Reyman, A. G., Semenov-Tian-Shansky, M. A., Frenkel, I. B.: Graded Lie algebras and completely integrable dynamical systems. *Sov. Math. Doklady*20, 811–814 (1979) · [Zbl 0437.58008](#)
- [19] Serre, J. P.: *Groupes algébriques et corps de classe*. Paris: Hermann,

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