

Drinfel'd, V. G.

Quasi-Hopf algebras. (English. Russian original) Zbl 0718.16033

Leningr. Math. J. 1, No. 6, 1419-1457 (1990); translation from Algebra Anal. 1, No. 6, 114-148 (1989).

The author defines the notion of quasi-Hopf algebra by weakening the coassociativity law for the coproduct in analogy to the weakening of the cocommutativity when passing from cocommutative to almost cocommutative Hopf algebras. A quasi-Hopf algebra is almost coassociative. The analogue of the R-matrix is an invertible element Φ of $A \otimes A \otimes A$, which is assumed to satisfy natural selfconsistency relations.

The paper is organized as follows: §1. Definition and elementary properties of quasi-Hopf algebras. The category mod A of left A -modules is discussed in case A denotes a quasi-bialgebra. There is an equivalence relation for quasi-bialgebras with different coproduct and different Φ , called "twisting". §2. Quasi-Lie bialgebras. The author defines and studies the "classical analogue" or classical limit, the quasi-Lie bialgebras corresponding to quasi-Hopf quantized universal enveloping algebras. §3. Quasitriangular, triangular, and coboundary quasi-Hopf algebras. In the case of quasitriangular, triangular, and coboundary quasi-Hopf algebras the category mod A is characterized by commutativity of some diagrams. The monoidal category mod A is symmetric or a tensor category if A is triangular. Quasitriangular quasi-Hopf quantized universal enveloping algebras are discussed together with their classical limits consisting of quasi-Lie bialgebras \mathfrak{g} endowed with a symmetric \mathfrak{g} -invariant element t of $\mathfrak{g} \otimes \mathfrak{g}$, t vanishes for triangular quasi-Hopf algebras. The classical limit of a coboundary quasi-Hopf quantized universal enveloping algebra is proved to be a quasi-Lie bialgebra \mathfrak{g} endowed with a \mathfrak{g} -invariant element ϕ of $\wedge^3 \mathfrak{g}$. ϕ vanishes in the triangular case. There is stated a one-to-one correspondence up to twisting between quasi-Hopf quantized universal enveloping algebras and the quasi-Lie bialgebras (\mathfrak{g}, t) endowed with a symmetric \mathfrak{g} -invariant tensor t , i.e. it is stated, that quasi-Lie algebras of that type can be quantized (Theorem 3.15). The idea of the proof is sketched in the introduction of the paper. The paper closes with a remark on the construction of link-invariants including R-matrix invariants for classical solutions of the quantum Yang-Baxter equation.

Reviewer: [H. Boseck \(Greifswald\)](#)

MSC:

- 16W30 Hopf algebras (associative rings and algebras) (MSC2000)
- 17B37 Quantum groups (quantized enveloping algebras) and related deformations
- 81R50 Quantum groups and related algebraic methods applied to problems in quantum theory
- 17B35 Universal enveloping (super)algebras

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coassociativity; almost cocommutative Hopf algebras; R-matrix; quasi-Hopf algebras; quasi-bialgebras; quasi-Lie bialgebras; quasi-Hopf quantized universal enveloping algebras; coboundary quasi-Hopf algebras; tensor categories; quasitriangular quasi-Hopf quantized universal enveloping algebras; link-invariants; quantum Yang-Baxter equation