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**Modules graded by G-sets.** (English) [Zbl 0721.16025](#)

*Math. Z.* 203, No. 4, 605-627 (1990).

Let  $R$  be an associative ring with unity graded by a group  $G$ . By a  $G$ -set we mean a set  $A$  with a  $G$ -action on  $A$ . A left  $R$ -module  $N$  is said to be graded by  $A$  if  $N = \bigoplus_{x \in A} N_x$  for some additive subgroups  $N_x$  such that  $R_g N_x \subseteq N_{gx}$  for  $g \in G, x \in A$ . The aim of the paper is to study the category  $(G, A, R)\text{-gr}$  consisting of the left  $R$ -modules graded by a  $G$ -set  $A$  with degree preserving  $R$ -linear maps as morphisms. The inspiration comes from the paper of *E. Dade* [*J. Reine Angew. Math.* 369, 40-86 (1986; [Zbl 0583.16001](#))], where some special cases of  $G$ -set gradations are applied to the Clifford theory of graded rings. In the first part of the paper the authors show that  $(G, A, R)\text{-gr}$  is a Grothendieck category. Then, the smash product construction  $R\#G$  for a ring graded by a finite group  $G$  is extended to the case  $R\#A$  of a finite  $G$ -set  $A$ . It is shown that  $(G, A, R)\text{-gr}$  is isomorphic to the category  $R\#A\text{-mod}$ . Moreover, a matrix characterization of  $R\#A$  is found. In the second part of the paper, certain functors introduced by Dade for  $G$ -sets of the form  $G/H$  (the left cosets of a subgroup  $H$  of  $G$  with  $G$  acting by left translations) are applied to general  $G$ -set gradations. These functors are particularly applied to the study of injective objects in  $(G, A, R)\text{-gr}$ .

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#### MSC:

[16W50](#) Graded rings and modules (associative rings and algebras)

[16D90](#) Module categories in associative algebras

[16S40](#) Smash products of general Hopf actions

[16D50](#) Injective modules, self-injective associative rings

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#### Keywords:

injective module;  $G$ -action; category; left  $R$ -modules;  $G$ -set gradations; Clifford theory; graded rings; Grothendieck category; smash product; functors

**Full Text:** [DOI](#)

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