Cohen, Albert; **Froment, Jacques**; **Istas, Jacques**

Analyse multirésolution des signaux aléatoires. (Multiscale analysis of random signal). (French) Zbl 0721.60048

C. R. Acad. Sci., Paris, Sér. I 312, No. 8, 567-570 (1991).

We consider a stationary random signal of the second order and we examine its multiresolution approximation in the sense given by *Y. Meyer* in [Ondelettes et opérateurs. Vol. I: Ondelettes (1990; Zbl 0694.41037), Vol. II: Opérateurs de Calderon-Zygmund (Paris, 1990)], i.e. its projections on a ladder of embedded spaces V_j representing its components at the scale 2^j. We are in particular interested in the mean square error between the signal and its approximation at a certain dyadic scale 2^j, and how this error evoluates asymptotically when the scale gets finer. This problem can be studied by using the wavelet basis ψ_k^j associated to the multiresolution analysis since these elements characterize at each scale the missing information between a level of approximation and the next finer level. By studying the expectation of the square modulus for each wavelet coefficient, we obtain a formula for the mean square error which involves both the wavelet which is used and the decay at infinity of the power spectrum for this random signal. Our result can be summarized in the following way: Suppose that the power spectrum decays at infinity like $|\omega|^{-p}$ and that the number of vanishing moments for the wavelet is superior to $(p+1)/2$, then the mean square error decays like $2^{(1-p)j}$ when j goes to $+\infty$.

This result can be viewed as the statistical version of the Hölder regularity characterization by the wavelet coefficients for deterministic function which is presented by Y. Meyer (loc. cit.) and *S. Jaffard* [C. R. Acad. Sci., Paris, Sér. I 308, No.4, 79-81 (1989; Zbl 0665.42012)]. It is indeed well-known that the wavelet coefficients of a smooth function decay at a geometric rate which is proportional to the degree of smoothness, provided that the wavelet has enough vanishing moments. Here the Hölder regularity is replaced by the "statistical regularity" expressed by the decay of the power spectrum.

Reviewer: J.Froment (Paris)

MSC:

60G35 Signal detection and filtering (aspects of stochastic processes)

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stationary random signal; mean square error; wavelet; power spectrum; Hölder regularity characterization; decay of the power spectrum