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Integrated semigroups and differential operators on L^p spaces. (English) Zbl 0724.34067
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In this note we prove that every operator A on $L^p(\mathbb{R}^n)$ whose symbol P is a purely imaginary elliptic polynomial on \mathbb{R}^n or is of the form $P(\xi) = i|\xi|^m$ generates an α -times integrated semigroup on $L^p(\mathbb{R}^n)$ ($1 < p < \infty$) whenever $\alpha \geq n|1/2 - 1/p|$ and an α -times integrated semigroup on $L^1(\mathbb{R}^n)$ or $L^\infty(\mathbb{R}^n)$ whenever $\alpha > n/2$. These constants are shown to be optimal for all powers of the Laplacian besides the square root. In that case A generates an α -times integrated semigroup ($\alpha > 0$) on $L^p(\mathbb{R}^n)$ ($1 < p < \infty$) if and only if $\alpha > (n-1)|1/2 - 1/p|$ and if and only if $\alpha > (n-1)/2$ in the cases $p = 1$ or $p = \infty$.

Reviewer: [M.Hieber](#)

MSC:

34G10 Linear differential equations in abstract spaces
42B15 Multipliers for harmonic analysis in several variables

Cited in **53** Documents

Keywords:

differential operator; symbol; integrated semigroup

Full Text: [DOI](#) [EuDML](#)

References:

- A1 Arendt, W.: Vector valued Laplace transforms and Cauchy problems. *Isr. J. Math.*59, 327-352 (1987). · [Zbl 0637.44001](#) · [doi:10.1007/BF02774144](#)
- A2 Arendt, W.: Sobolev imbeddings and integrated semigroups. Preprint 1990
- A.K Arendt, W., Kellermann, H.: Integrated solutions of Volterra integro-differential equations and applications. In: Da Prato, G., Iannelli, M. (eds.). *Volterra integrodifferential equations in Banach spaces and applications*. Proc. Conf. Trento 1987, 21-51, Pitman Res. Notes Math. Ser., 190. Harlow: Longman 1989
- B, E Balabane, M., Emamirad, H.A.: L^p estimates for Schrödinger evolution equations. *Trans. Am. Math. Soc.*291, 357-373 (1985) · [Zbl 0588.35029](#)
- F, StxxxFefferman, C., Stein, E.M.: H^p spaces of several variables. *Acta Math.*129, 137-193 (1972). · [Zbl 0257.46078](#) · [doi:10.1007/BF02392215](#)
- G Goldstein, J.A.: *Semigroups of linear operators and applications*. New York: Oxford University Press 1985 · [Zbl 0592.47034](#)
- Hi1 Hieber, M.: Laplace transforms and α -times integrated semigroups. To appear in *Forum Math.*
- Hi2 Hieber, M.: Integrated semigroups and the Cauchy problem for systems in L^p -spaces. to appear in *J. Math. Anal. Appl.*
- H?1 Hörmander, L.: Estimates for translation invariant operators in L^p spaces. *Acta Math.*104, 93-140 (1960) · [Zbl 0093.11402](#) · [doi:10.1007/BF02547187](#)
- H?2 Hörmander, L.: *The analysis of linear partial differential operators. I*. Berlin Heidelberg New York: Springer 1984
- K, HiKellermann, H., Hieber, M.: Interated semigroups. *J. Funct. Anal.*84, 160-180 (1989) · [Zbl 0689.47014](#) · [doi:10.1016/0022-1236\(89\)90116-X](#)
- La Lanconelli, E.: Valutazioni in L^p ($p \geq 1$) della soluzione del problema di Cauchy per l'equazione di Schrödinger. *Boll. Un. Mat. Ital.*4, 591-607 (1968) · [Zbl 0167.10401](#)
- Li Littman, W.: The wave operator and L^p -norms. *J. Math. Mech.*12, 55-68 (1963) · [Zbl 0127.31705](#)
- M1 Miyachi, A.: On some Fourier multipliers for H^p ($p \geq 1$). *J. Fac. Sci. Univ. Tokyo*27, 157-179 (1980) · [Zbl 0433.42019](#)
- M2 Miyachi, A.: On some estimates for the wave equation in L^p and H^p . *J. Fac. Sci. Univ. Tokyo*27, 331-354 (1980) · [Zbl 0437.35042](#)
- M3 Miyachi, A.: On some singular Fourier multipliers. *J. Fac. Sci. Univ. Tokyo*28, 267-315 (1981) · [Zbl 0469.42003](#)
- N Neubrandner, F.: Integrated semigroups and their applications to the abstract Cauchy problem. *Pacific J. Math.*135, 111-155 (1988) · [Zbl 0675.47030](#)
- P Peral, J.C.: L^p estimates for the wave equation. *J. Funct. Anal.*36, 114-145 (1980) · [Zbl 0442.35017](#) · [doi:10.1016/0022-1236\(80\)90110-X](#)
- Sj Sjöstrand, S.: On the Riesz means of the solution of the Schrödinger equation. *Ann. Scuola Norm. Sup. Pisa*24, 331-348 (1970)

· [Zbl 0201.14901](#)

St Stein, E.M.: Singular integrals and differentiability properties of functions. New Jersey: Princeton University Press 1970 · [Zbl 0207.13501](#)

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