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The kernel of an automorphic derivation and an application to normal subfields of generalized André-systems. (Der Kern einer automorphen Ableitung und eine Anwendung auf normale Teilkörper verallgemeinerter André-Systeme.) (German) Zbl 0725.12006

Arch. Math. 58, No. 5, 514-520 (1992).

Let Q be a quasifield and K a subfield (not necessary commutative) of the kernel of Q. A map $\phi : Q^* \to \mathrm{GL}(Q,K)$; $a \to \phi_a$ ($Q^* := Q \setminus \{0\}$) is called a derivation, if the derived quasifield $Q^\phi := (Q,+,\circ)$ is a quasifield as well. Here $a \circ b := a\phi_a(b)$, $a \neq 0$ and $0 \circ b := 0$. The subgroup of $\mathrm{GL}(Q,K)$ generated by $\phi(Q^*)$ is denoted by Δ_ϕ. If Q is a (skew)field and Δ_ϕ is contained in $\mathrm{Aut}(Q)$, then ϕ is called automorphic and Q^ϕ is usually named a generalized André-system. The kernel of ϕ is the set $\mathrm{Ker}\,\phi := \{a \in Q^*;\ \phi_{ax} = \phi_x\}$, and is a subgroup of Q^*.

This idea implicitly has been used before [*D. A. Foulser*, Math. Z. 100, 380–395 (1967; Zbl 0152.18903); *A. Herzer*, Arch. Math. 52, No. 1, 99–104 (1989; Zbl 0633.51003)]. It proved very useful in the study of the structure of generalized André-systems.

Some relations between $\mathrm{Ker}\,\phi$ and the nuclei, the fixed field of Δ_ϕ and the center of the derived quasifield are given. These results are used to generalize theorems on normal subfields of nearfields [cf. *H. Wähling*, Theorie der Fastkörper. Essen: Thales Verlag (1987; Zbl 0669.12014), (III.5.5)]. In the last section some examples are given.

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