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Rectified homotopical depth and Grothendieck conjectures. (English) Zbl 0725.14016

The Grothendieck Festschrift, Collect. Artic. in Honor of the 60th Birthday of A. Grothendieck. Vol. II, Prog. Math. 87, 311-351 (1990).

[For the entire collection see [Zbl 0717.00009](#).]

Let $\emptyset \neq X$ be a reduced complex analytic space with \mathcal{S} a Whitney stratification on X , where X_i denotes the union of strata of dimension $\leq i$. The rectified homotopical depth $rhds_{\mathcal{S}}(X)$ of X is $\geq n$ if, for any i and any point $x \in X_i \setminus X_{i-1}$, there exists a fundamental system (U_{α}) of neighbourhoods of x in X such that, for any α the pair $(U_{\alpha}, U_{\alpha} \setminus X_i)$ is $(n-1-i)$ -connected. Then $rhds_{\mathcal{S}}(X)$ is defined as the maximum of the integers n such that $rhds_{\mathcal{S}}(X) \geq n$. This definition is shown to be equivalent to the ones of A. Grothendieck [see Séminaire de géométrie algébrique, SGA 2 (1962; [Zbl 0159.504](#)); Exposé XIII, p. 27, definition 2; see also Adv. Stud. Pure Math. 2 (1968; [Zbl 0197.472](#))]. Replacing the connectedness condition by the vanishing of $H_k(U_{\alpha}, U_{\alpha} \setminus X_i, \mathbb{Z})$ for any $k < n - i$ the authors define the rectified homological depth of X . - A. Grothendieck conjectured that the notion of rectified homotopical (resp. homological) depth gives the right level of comparison for the homotopy (resp. homology) type between X and a hyperplane section, as stated in theorems of Lefschetz type for nonsingular varieties.

The authors give positive answers to Grothendieck's conjectures. The proofs become possible because of a handy formulation of the notion of rectified homotopical depth using Whitney stratifications shown in the paper. More general there are Lefschetz type theorems for open varieties replacing the hyperplane section by a good neighbourhood of a hyperplane section as has been done by P. Deligne [see Sémin. Bourbaki, 32e année, Vol. 1979/80, Exposé 543, Lect. Notes Math. 842, 1-10 (1981; [Zbl 0478.14008](#))].

Reviewer: [P.Schenzel \(Halle\)](#)

MSC:

- [14F35](#) Homotopy theory and fundamental groups in algebraic geometry
- [14D05](#) Structure of families (Picard-Lefschetz, monodromy, etc.)
- [32C15](#) Complex spaces
- [14F05](#) Sheaves, derived categories of sheaves, etc. (MSC2010)

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Keywords:

[Lefschetz theorems](#); [complex analytic space](#); [Whitney stratification](#); [rectified homotopical depth](#); [hyperplane section](#)