Kud'yavin, V. S.
A characteristic property of a class of space homeomorphisms. (Russian) [Zbl 0725.30010]

[For the entire collection see Zbl 0685.00003.]
This is a short note on particular properties of some classes of homeomorphisms \( f: D \to D^* \) between bounded open sets in \( \mathbb{R}^n \). A pair \((E,G)\) consisting of an open set \( G \) in \( \mathbb{R}^n \) and a subset \( E \subset G \) compact in \( \mathbb{R}^n \) is called a condensator. For any \( \alpha \) with \( 1 \leq \alpha \leq n \), such a condensator has a real- valued \( \alpha \)-capacity which is defined by an infimum of integrals over \( G \), taken over a certain class of functions \( \phi : G \to [0,1] \).

Theorem 1 of this paper says that a homeomorphism \( f: D \to D^* \) is almost everywhere differentiable in \( D \) if \( f \) satisfies certain inequalities with respect to the \( \alpha \)-capacities of condensators \((E,G)\) in \( D \), i.e., \( E \subset G \subset D \). The second part of the paper deals with characteristic values attached to homeomorphisms \( f: D \to D^* \), which are defined by particular integrals (over \( D \)) of functions constructed from (pointwise) generalized derivatives of \( f \). Theorem 2 gives then a characterization of those homeomorphisms that have prescribed bounded characteristic integrals in the above sense. The equivalent conditions are expressed in terms of (then existing) quasi-additive set-valued functions on \( D \), which satisfy boundedness conditions for condensators in \( D \).

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MSC:
- 30C65 Quasiconformal mappings in \( \mathbb{R}^n \), other generalizations
- 26D10 Inequalities involving derivatives and differential and integral operators
- 26E25 Set-valued functions
- 54C30 Real-valued functions in general topology

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homeomorphisms of real domains; capacity; differentiability; nondegenerate functions; generalized derivatives; set-valued functions