Chui, Charles K.; Lai, Mingjun

**On bivariate super vertex splines.** (English) Zbl 0726.41012

Constructive Approximation 6, No. 4, 399-419 (1990).

The authors construct a vertex spline basis for the super spline subspace $\hat{S}^r_d(\Delta)$ of $S^r_d(\Delta)$. Here $\Delta$ means an arbitrary regular triangulation in $\mathbb{R}^2$,

$$S^{r,\ell}_d(\Delta) = \{s \in S^r_d(\Delta),\ D^\alpha s(v) \text{ exists for } |\alpha| \le \ell \text{ and every vertex } v \text{ of } \Delta\}$$

and a vertex spline of $\hat{S}^r_d(\Delta) = S^{r,r+\lfloor(d-2r-1)/2\rfloor}_d(\Delta)$ has a support which contains at most one vertex of $\Delta$ in its interior. *C. de Boor* and *K. Höllig* [Math. Z. 197, 343-363 (1987; Zbl 0616.41010)] proved that $S^r_d(\Delta)$ has approximation order $d+1$ provided that $d \ge 3r+2$. Here it is shown that this can be achieved already by using avertex spline basis. Therefore a quasi-interpolatory linear operator L is considered which reproduces functions from $\hat{S}^r_{3r+2}(\Delta)$. Then for $d \ge 3r+2$ and sufficiently smooth functions f it holds $\|f - Lf\| \le C\|D^{d+1}f\||\Delta|^{d+1}$. The proofs are based on the Bernstein-Bézier technique, which gives a very constructive approach to the vertex splines.

Reviewer: J.Prestin (Rostock)

**MSC:**

41A15 Spline approximation
41A25 Rate of convergence, degree of approximation
41A63 Multidimensional problems (should also be assigned at least one other classification number from Section 41-XX)



**Keywords:**

super spline; Bernstein-Bézier technique; vertex splines

**Full Text:** DOI

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