Perthame, Benoit; **Tadmor, Eitan**

A kinetic equation with kinetic entropy functions for scalar conservation laws. (English) Zbl 0729.76070

Commun. Math. Phys. 136, No. 3, 501-517 (1991).

The authors consider nonlinear kinetic equation of the type

$$[\partial_t + a(v)\cdot\partial_x]f_\epsilon(x,v,t) = \frac{1}{\epsilon}[\chi_{u_\epsilon(x,t)}(v) - f_\epsilon(x,v,t)],$$

where $x\in\mathbb{R}^d$, $v\in\mathbb{R}$, $t\in\mathbb{R}_+$, $u_\epsilon(x,t)=\int f_\epsilon(x,v,t)dv$ and

sgn u if $(u-v)=0$ $\chi_u(v)=0$ otherwise.

This model resembles the well-known BGK model. It is proved that the initial value problem is uniformly in ϵ well-posed (i.e. bounds and continuous dependence on the data hold uniformly with respect to ϵ). Moreover, the kinetic equation possesses a family of kinetic entropy functions, which translate as $\epsilon\to 0$ to Krushkov-type entropy inequalities for the corresponding multidimensional conservation law

$$\partial_t u(x,t) + \sum_{i=1}^d \partial_x[A_i(u(x,t))] = 0,$$

where

$$a(v)\cdot\partial_x \equiv \sum_{i=1}^d a_i(v)\partial_{x_i},\ \text{ and } a_i(\cdot)=A_i'(\cdot).$$

Finally, bounded variation arguments in the multidimensional case and a compensated compactness argument in the one- dimensional case are used to prove that the local density u_ϵ of f_ϵ converges strongly with $\epsilon\searrow 0$ to the unique entropy solution of the conservation law.

Reviewer: R.Illner (Victoria)

MSC:

76P05 Rarefied gas flows, Boltzmann equation in fluid mechanics
82C40 Kinetic theory of gases in time-dependent statistical mechanics
35L65 Hyperbolic conservation laws

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