Levasseur, Thierry

Relèvements d'opérateurs différentiels sur les anneaux d'invariants. (Liftings of differential operators over rings of invariants). (French) Zbl 0733.16009

Operator algebras, unitary representations, enveloping algebras, and invariant theory, Proc. Colloq. in Honour of J. Dixmier, Paris/Fr. 1989, Prog. Math. 92, 449-470 (1990).

[For the entire collection see Zbl 0719.00018.]

Let $G \to GL(V)$ be a finite dimensional representation of the reductive group G, and let V//G denote the affine variety whose ring of regular functions $\mathcal{O}(V//G)$ equals $\mathcal{O}(V)^G$. The group G operates naturally on the ring $\mathcal{D}(V)$ of differential operators on V and maps each of the $\mathcal{O}(V)$-modules $\mathcal{D}er^m(V)$ of differential operators of order $\leq m$ with zero constant term into itself. Then (V,G) is said to have the property of lifting to order m if the restriction morphism $\phi : \mathcal{D}er^m(V)^G \to \mathcal{D}er^m(V//G)$ is surjective. If this holds for all m, then (V,G) is said to satisfy the lifting property, and this will imply the surjectivity of $\phi : \mathcal{D}(V)^G \to \mathcal{D}(V//G)$. The author provides a sufficient condition for this to happen. In order to formulate his main result, let $\mathcal{A}(V//G)$ denote the ideal of $\mathcal{O}(V)^G$ defining the complement of the principal stratum of V//G, let gr ϕ :gr $\mathcal{D}(V)^G \to gr\mathcal{D}(V//G)$ be the graded morphism associated with ϕ, and set $R = gr\mathcal{D}(V)^G/\ker(gr\phi)$. Then, if R is the intersection of all localizations of its height one primes, and if $\mathcal{A}(V//G)$ generates an ideal of height ≥ 2 in R, then (V,G) has the lifting property. Among other applications, this is used to treat the cases when G is finite or the group of nonzero complex numbers, since in both instances the structure of R is fairly well known.

Reviewer: G.Krause (Winnipeg)

MSC:

16S32	Rings of differential operators (associative algebraic aspects)
16W50	Graded rings and modules (associative rings and algebras)
20G05	Representation theory for linear algebraic groups
14A10	Varieties and morphisms
14L30	Group actions on varieties or schemes (quotients)
13N10	Commutative rings of differential operators and their modules

Keywords:

ring of differential operators; finite dimensional representation; reductive group; affine variety; ring of regular functions; lifting property; graded morphism; height one primes