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On a superlinear elliptic equation. (English) Zbl 0733.35043
Ann. Inst. Henri Poincaré, Anal. Non Linéaire 8, No. 1, 43-57 (1991).

The author deals with the following problem:

$$-\Delta u = f(u) \text{ in } \Omega, \quad u = 0 \text{ on } \partial\Omega,$$

where Ω is a bounded domain in R^n with regular boundary, assuming that

(f1) $f \in C^1(R, R)$, $f(0) = f'(0) = 0$;

(f2) There are constants C_1, C_2 such that

$$|f(t)| \leq C_1 + C_2|t|^\alpha, \quad 1 < \alpha < (n+2)/(n-2)$$

(f3) There are constants $\mu > 2$, $M > 0$ such that

$$0 < \mu F(t) \leq tf(t), \quad |t| \geq M, \text{ where } F(t) = \int_0^t f(r)dr.$$

The main result is

Theorem. If f satisfies (f1)(f2)(f3), then the problem above possesses at least three nontrivial solutions.

In a classical paper, Ambrosetti and Rabinowitz obtained two nontrivial solutions, and infinitely many in the case of odd nonlinearities f . Infinitely many solutions can be obtained in case $n = 1$. The author establishes existence of multiple solutions in case $n \geq 2$ without assuming any symmetry.

Reviewer: J.E.Bouillet (Buenos Aires)

MSC:

35J65 Nonlinear boundary value problems for linear elliptic equations
58E05 Abstract critical point theory (Morse theory, Lyusternik-Shnirel'man theory, etc.) in infinite-dimensional spaces

Cited in **4** Reviews
Cited in **88** Documents

Keywords:

superlinear; subcritical growth; Dirichlet problem; three nontrivial solutions

Full Text: [DOI](#) [Numdam](#) [EuDML](#)

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