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Quantum spectral transform method. Recent developments. (English) Zbl 0734.35071

Integrable quantum field theories, Proc. Symp., Tvärminne, 1981, Lect. Notes Phys. 151, 61-119 (1982).

[For the entire collection see Zbl 0488.00024.]

The present lectures are devoted to the most promising, from our viewpoint, direction in studying the quantum completely integrable models, the so-called quantum spectral transform method (QSTM; other names are the quantum inverse scattering method, the quantum inverse problem method). The QSTM was developed 3 years ago as the result of a synthesis of two major directions in the modern theory of exactly soluble systems. The first of them is based on the tradition of studying the exactly soluble models of solid state and statistical physics which originates from the works of H. Bethe and L. Onsager and culminates in works of R. I. Baxter. The second direction mentioned is the classical spectral transform method (CSTM), which yielded many important results in the theory of the classical completely integrable systems. The most impressive achievements of the QSTM are: 1. The solution of the sine- Gordon model, i.e. finding its mass spectrum and S-matrix. 2. The solution of the quantum inverse problem for the nonlinear Schrödinger equation, which enables one to reproduce the known results for the Green's functions of the one-dimensional impenetrable Bose gas. The next problem which is likely to be solved in the near future is the calculation of Green's functions for the one-dimensional Bose gas with δ-function interaction. 3. In the purely methodical respect, the QSTM has given completeness and transparency to the conventional Bethe ansatz technique. In contrast to the old (coordinate) Bethe ansatz, the new (algebraic) Bethe ansatz allows one to obtain the eigenvalues of the Hamiltonian and other integrals of motion immediately, omitting the stage which is necessary in the conventional approach, of tediously investigating the eigenfunctions in their manifest coordinate representation. We now outline the plan of the paper in more detail. Section 2 presents a general survey of the QSTM and of the models solved via the QSTM. Subsections 2.1 and 2.2 introduce the reader to the QSTM using the nonlinear Schrödinger equation as the example. In Subsection 2.3 the QSTM is applied to the massive Thirring model both in Bose and Fermi cases. Subsection 2.4 generalizes the method to the lattice models. In Subsection 2.5 a general scheme of applying the QSTM is described. Subsection 2.6 contains the list of models which are solved and are to be solved via the QSTM. Sections 3 and 4 deal with the so-called Yang-Baxter equation (YBE) which is shown to have many applications in the theory of exactly soluble classical and quantum systems. In Section 3 the origin and the applications of the YBE are described. In Section 4 a method of constructing new solutions to the YBE is discussed which is based on the algebraic approach proposed by the authors. The method itself is described in Subsection 4.1, and Subsection 4.2 contains discussion of a new class of exactly soluble models which are generated by the solutions to YBE. In Section 5 we discuss the problem of generalizing the notion of the transition matrix determinant to the quantum case. This arises in applying the QSTM to models including several fields.

MSC:

35P25 Scattering theory for PDEs
35R30 Inverse problems for PDEs
81U40 Inverse scattering problems in quantum theory
35-02 Research exposition (monographs, survey articles) pertaining to partial differential equations
81-02 Research exposition (monographs, survey articles) pertaining to quantum theory
35Q55 NLS equations (nonlinear Schrödinger equations)
35Q58 Other completely integrable PDE (MSC2000)
35Q53 KdV equations (Korteweg-de Vries equations)
81Q99 General mathematical topics and methods in quantum theory

Keywords:

quantum completely integrable; quantum spectral transform method; quantum inverse scattering method;

quantum inverse problem method; sine-Gordon model; nonlinear Schrödinger equation; Bethe ansatz; eigenvalues; Hamiltonian; Yang-Baxter equation