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Transformation of measure on Wiener space. (English) Zbl 0938.46045

Springer Monographs in Mathematics. Berlin: Springer. xiii, 297 p. DM 159.00; öS 1161.00; sFr. 144.00; £61.00; $ 96.00 (2000).

This book deals with transformation of Gaussian measures on infinite-dimensional Banach spaces. There are actually two approaches to this subject. The first, concerning the classical Wiener measure, grew out of the work of Cameron and Martin in the 1940's, who studied perturbations of Wiener measure by deterministic paths with finite energy. Their work was extended by Girsanov in the 1960's to the case of random adapted perturbations of Wiener measure. Girsanov's work played a central role in stochastic analysis and stochastic control theory in the coming decades. The subject of transformation of Wiener measure received fresh impetus with the development of the Malliavin calculus in the mid-seventies. This provided a framework for the construction of stochastic integrals with anticipating integrands (these had earlier been introduced by Skorohod) and led to the study of non-adapted transformations of Wiener measure. The latter subject was also motivated by problems in stochastic partial differential equations and mathematical finance where one studies stochastic differential equations with terminal, as opposed to initial, conditions. Results addressing non-adapted transformations of classical Wiener measure were obtained by Buckdahn, Nualart-Pardoux, Nualart-Zakai, Zakai-Ustunel, Protter, and others.

The second approach to the problem of transformation of Gaussian measure originated in the theory of abstract Wiener spaces pioneered by Gross in 1965. In this work Gross showed how to construct a Gaussian measure μ on an abstract Banach space E as the completion of the canonical Gaussian cylinder set measure on a densely embedded underlying Hilbert space H. He later studied H-differentiable functions on E, an idea that can be considered as a precursor of the criterion of differentiability for Wiener functionals later introduced by Malliavin. Following Gross' work, Kuo gave conditions under which the transformation of μ under a map T of E of the form $I+K$, where K is a smooth map into E^* (identified in a natural way with a subset of E) gives rise to a measure $T(\mu)$ absolutely continuous with respect to μ. The work of Gross was later extended by Kuo.

A breakthrough in this area came with a paper of Ramer who showed how to obtain the same conclusion under fundamentally weaker hypotheses which allow K to map into H as opposed to E^*. A novel feature of Ramer's work is his formula for the Radon-Nikodým derivative $dT(\mu)/d\mu$. Two of the terms that occur in the corresponding formula of Kuo can fail to exist under Ramer's hypothesis; they appear in Ramer's formula as the the limit of the difference of two possibly divergent divergent sequences of random variables. Ramer's work was extended by Kusuoka. In the early eighties, Cruzeiro and the reviewer independently introduced another approach to the study of transformation of abstract Gaussian measures, based on embedding the transforming map T into a one parameter family. Further results along this line by Bogachev, Daletskii, and Smolyanov-Weizsäcker followed.

The book by Ustunel and Zakaii give an account of these developments. Chapter 1 provides background material and preliminary results. Chapter 2 discusses adapted perturbations of the Wiener path and the transformations of the Wiener measure they induce, and presents the Girsanov theorem. Chapters 4 and 5 describe results on the transformation of abstract Wiener measure. Chapter 6 addresses the transformation of measure under a special class of shifts, termed monotone shifts. An interesting distinction between the two approaches to the transformation of Wiener mesure described above is the different nature of the hypotheses assumed in each case. In the "classical" approach one assumes adaptedness of the transforming perturbation but does not require it to be smooth as a function of the Wiener path. The reverse is the case in the "abstract" approach. In Chapter 7, the authors attempt to bridge this gap. They address the problem of defining a Radon-Nikodým derivative in the abstract context, in cases where smoothness assumptions do not hold. Chapter 8 deals with the transformation of Wiener measure under rotations. Finally, Chapter 9 is devoted to the degree theory for Wiener space, as introduced by Getzler.

The book contains two appendices, the first giving some basic inequalities and the second giving a brief introduction to the Malliavin calculus, a subject that has turned out to be closely connected with the later development of the measure transformation problem.

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MSC:

46G12	Measures and integration on abstract linear spaces
46-02	Research exposition (monographs, survey articles) pertaining to functional analysis
28-02	Research exposition (monographs, survey articles) pertaining to measure and integration
60-02	Research exposition (monographs, survey articles) pertaining to probability theory
60H05	Stochastic integrals
60G15	Gaussian processes
60G30	Continuity and singularity of induced measures
28C20	Set functions and measures and integrals in infinite-dimensional spaces (Wiener measure, Gaussian measure, etc.)
60H25	Random operators and equations (aspects of stochastic analysis)
60G35	Signal detection and filtering (aspects of stochastic processes)
46T12	Measure (Gaussian, cylindrical, etc.) and integrals (Feynman, path, Fresnel, etc.) on manifolds
60H07	Stochastic calculus of variations and the Malliavin calculus

Keywords:

transformation of Wiener measure; abstract Wiener space; stochastic control theory; Malliavin calculus; canonical Gaussian cylinder set measure; Radon-Nikodym derivative; Ramer's formula; Girsanov theorem