The study of “instantons,” i.e. self-dual finite energy solutions of the Yang-Mills equations on Euclidean 4-space has developed rapidly since the initial examples was given by 't Hooft, Belavin ard Polyakov. Thnis paper marks the end of the first phase, for it shows that every solution can be obtained by a quaternionic matrix construction. Despite the fact that these defining matrices are still difficult to parametrize explicitly, the formalism has since yielded algebraic expressions for the scalar Green’s function and the Dirac fields in the Yang-Mills background. The proof of completeness of the construction passes from Euclidean space to its conformal compactification the 4-sphere $S^4$ and thence by Penrose’s twistors to complex projective 3-space $\mathbb{P}^3(\mathbb{C})$. Using the recent work of Barth and Horrocks in algebraic geometry, and a crucial vanishing theorem, the result follows. The vanishing theorem itself uses the relationship between sheaf cohomology groups on $\mathbb{P}^3(\mathbb{C})$ and the conformally invariant Laplacian on $S^4$, another aspect of twistor theory.

Reviewer: Michael Atiyah

For a scan of this review see the web version.

MSC:

14D21 Applications of vector bundles and moduli spaces in mathematical physics (twistor theory, instantons, quantum field theory)
14F06 Sheaves in algebraic geometry
14J60 Vector bundles on surfaces and higher-dimensional varieties, and their moduli
81T08 Constructive quantum field theory
81T13 Yang-Mills and other gauge theories in quantum field theory
53C80 Applications of global differential geometry to the sciences

Keywords:
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References:

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