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The perimeter inequality under Steiner symmetrization: cases of equality. (English)

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The main property of perimeter in connection with Steiner symmetrization is that if E is any set of finite perimeter $P(E)$ in $\mathbb{R}^n$, $n \geq 2$, and H is any hyperplane, then also its Steiner symmetral E^S about H is of finite perimeter and

$$P(E^S) \leq P(E).$$

This paper presents a characterization of the sets whose perimeter is preserved under this symmetrization. It is possible to assume without loss of generality that $H = \{(x', 0) : x' \in \mathbb{R}^{n-1}\}$. Let Ω denote an open set in $\mathbb{R}^{n-1}$. The authors find the following minimal assumptions to ensure the equivalence (up to translation) between sets E and E^S such that $P(E^S) = P(E)$: 1) $\partial^* E^S$ cannot have flat parts along the y-axis in $\Omega \times \mathbb{R}$ with outer $(n-1)$-dimensional measure strictly positive. (∂^* denotes the reduced boundary operator). 2) No (too large) subset of $E^S \cap (\Omega \times \mathbb{R})$ shrinks along the y-axis enough to be contained in $\Omega \times \{0\}$. The authors also provide a local symmetry result for E on any cylinder parallel to the y-axis having the form $\Omega \times \mathbb{R}$. The proofs involve quite subtle matters requiring delicate tools from geometric measure theory.

Reviewer: Salvador Gomis (Alicante)

MSC:

28A75	Length, area, volume, other geometric measure theory
26B15	Integration of real functions of several variables: length, area, volume
49Q15	Geometric measure and integration theory, integral and normal currents in optimization
52A38	Length, area, volume and convex sets (aspects of convex geometry)
52A40	Inequalities and extremum problems involving convexity in convex geometry
26D20	Other analytical inequalities

Keywords:

Steiner symmetrization; perimeter; Hausdorff measure; connectedness; geometric measure theory; Lebesgue measure; Lebesgue representative; Lebesgue points of Sobolev functions

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