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Elsenhans, Andreas-Stephan; Jahnel, Jörg
The Diophantine equation $x^4 + 2y^4 = z^4 + 4w^4$. (English) Zbl 1138.11056
Math. Comput. 75, No. 254, 935-940 (2006).

In the "Workshop on Rational and Integral Points on Higher-Dimensional Varieties" held in Palo Alto CA (2002), Sir *P. Swinnerton-Dyer* posed the following problem: "Does there exist a $K3$ surface S over $\mathbb{Q}$ such that $0 < \#S(\mathbb{Q}) < \infty$?" [Problem/Questions 6a; Boston: Birkhäuser Prog. Math. 226, 235–257 (2004; Zbl 1211.11077)]. One possible candidate for a $K3$ surface with the above property is the projective surface defined by the equation $x^4 + 2y^4 = z^4 + 4w^4$.

It has the $\mathbb{Q}$-rational points (1:0:1:0) and (1:0:−1:0). Sir P. Swinnerton-Dyer posed also the problem to find a third rational point on this surface [Problem/Questions 6c (loc. cit.)].

The paper under review gives an answer to this problem. More precisely, a systematic search by computer, shows that the projective surface defined by $x^4 + 2y^4 = z^4 + 4w^4$ admits precisely ten $\mathbb{Q}$-rational points which allow integral coordinates within the hypercube $|x|, |y|, |z|, |w| < 2,5 \times 10^6$. These are the points (±1:0:±1:0), (±1484801:±1203120:±1169407:±1157520).

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MSC:

11Y50 Computer solution of Diophantine equations
11D25 Cubic and quartic Diophantine equations
14G05 Rational points

Keywords:

$K3$ surface; diagonal quartic surface; rational point; Diophantine equation; computer solution

Software:

ARIBAS

Full Text: DOI

References:

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paper as accurately as possible without claiming the completeness or perfect precision of the matching.