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On a regularity criterion in terms of the gradient of pressure for the Navier-Stokes equations in $\mathbb{R}^N$. (English) Zbl 1099.35091

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The Cauchy problem for the Navier-Stokes equations is considered in $\mathbb{R}^n \times (0,T)$, $n = 3, 4$

$$\frac{\partial v}{\partial t} + (v \cdot \nabla)v - \Delta v + \nabla p = 0, \quad \operatorname{div} v = 0 \quad \text{in} \quad \mathbb{R}^n \times (0,T)$$
$$v(x,0) = v_0(x), \quad x \in \mathbb{R}^n$$

Let

$$v_0 \in L_2(\mathbb{R}^n) \bigcap L_q(\mathbb{R}^n) \quad \text{for } q \geq n, \quad \operatorname{div} v_0 = 0.$$

It is proved if v is a Leray-Hopf weak solution to the problem and

$$\nabla p \in L_\alpha\left(0,T; L_\gamma(\mathbb{R}^n)\right) \quad \text{with } \frac{2}{\alpha} + \frac{n}{\gamma} \leq 3,\ \frac{2}{3} < \alpha < \infty,\ \frac{n}{3} < \gamma < \infty$$

then v is regular and unique. A priori estimates for the smooth solution are the base of the proof.

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MSC:

35Q30	Navier-Stokes equations
76D03	Existence, uniqueness, and regularity theory for incompressible viscous fluids
76D05	Navier-Stokes equations for incompressible viscous fluids
35B45	A priori estimates in context of PDEs
35D10	Regularity of generalized solutions of PDE (MSC2000)

Keywords:

regularity; a priori estimates; Leray-Hopf weak solution

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