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Quantum finance. Path integrals and Hamiltonians for options and interest rates. (English)

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This book is for physicists and mathematicians working in the field of finance, to quantitative analysts of fixed income and foreign exchange. It can also be used as textbook for courses in financial physics and mathematics. The mathematical background of the reader should be a good grasp of calculus, familiar with linear algebra, and a good knwledge of probability theory.

The aim of this book is to apply the mathematics and conceptional formalism of quantum mechanics and quantum field theory of physics to the theory of options and the modelling of interest rates of economics. It is contrary to the classical financial mathematics dominated by stochastic calculus. Options are expressed and solved by Hamiltonians and path integral. Nonlinear processes such as stock options with stochastic volatility can be solved using path integration.

This book has 3 parts with 11 chapters. After the synopsis (chapter 1), part I begins with the introduction to finance and derivative securities. In part II, Hamiltonians and path integrals are applied to the study of stock options and stochastic interest rates models. In part III, the forward interest rates are regarded as quantum field.

Part I (Fundamental Concepts of Finance) has 2 chapters. Chapter 2 introduces to finance. Fundamental concepts and terminology of finance, in particular risk, return, time value of money, arbitrage, hedging are discussed. Chapter 3 introduces the concept of financial derivatives and discusses the pricing of derivatives, e.g. the Black and Scholes model, the Merton-Garman equation.

In part II (Systems with Finite Numbers of Degrees of Freedom), the degree of freedom is defined as the number of independent random variables at each instant of time t as for example the spot rate $r(t)$ with one degree of freedom. Chapter 4 treats the problem of pricing derivative securities as a problem of quantum mechanics and the Hamiltonians driving the price of options are derived for stock prices and stochastic volatility. Chapter 5 expresses the problem of option pricing as a Feynman path integral. The Hamiltonians provide a link between the partial differential equations of option pricing and its path-integral realization. In chapter 6, some of the important stochastic models for the spot and forward interest rates are reviewed. It prepares for the main thrust of this book namely the application of quantum field theory to the modelling of interest rates.

In part III (The Quantum Field Theory of Interest Rates), quantum field theory is a mathematical structure for studying systems with infinitely many degrees of freedom. All chapters in this part regard the forward interest rates as a quantum field. Chapter 7 describes the formalism of path integration and applies it to a randomly evolving curve. The forward interest rates are modelled as a randomly fluctuating curve described by quantum field theory. Various models are studied to illustrate the theoretical flexibility of this approach. Chapter 8 discusses the empirical aspects of the forward interest rates in some details and shows how to calibrate and test field theory models. The most important result of this chapter is that the "stiff" Gaussian field theory model provides an almost exact fit for the market behaviour of the forward rates. Chapter 9 studies the pricing and hedging of Treasury Bond futures, Bond options, and interest caps. Chapter 10 derives the state space and Hamiltonians for linear and nonlinear theories. The Hamiltonian formulation yields an exact solution of the martingale condition for the nonlinear forward rates and the forward rates with stochastic volatility. Chapter 11 (Conclusions) gives a short summary of the main ideas of this book.

Reviewer: Klaus Ehemann (Karlsruhe)

MSC:

91B28	Finance etc. (MSC2000)
81P99	Foundations, quantum information and its processing, quantum axioms, and philosophy
91-02	Research exposition (monographs, survey articles) pertaining to game theory, economics, and finance

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