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Local rigidity of 3-dimensional cone-manifolds. (English) Zbl 1098.53038

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Cone-manifolds can be viewed as generalizations of geometric orbifolds, where the cone-angles are no longer restricted to the set of orbifold-angles, which are rational multiples of π. In particular, it is known after W. P. Thurston that 3-dimensional cone-manifolds arise naturally in the geometrization of 3-dimensional orbifolds. In this paper, the author studies the local deformation space of 3-dimensional cone-manifold structures of constant curvature $\kappa \in \{-1, 0, 1\}$, and cone-angles less than or equal to π. In the hyperbolic and in the spherical cases his main result is a vanishing theorem for $H^1_{L^2}(M; \mathcal{E})$, the first L^2-cohomology group of the smooth part $M = C \setminus \Sigma$ of the cone-manifold C (Σ being the singular part of C), with coefficients in the flat bundle of infinitesimal isometries.

From this result, he can conclude local rigidity (in the spherical case it is assumed that C is not Seifert fibered): the set of cone-angles $\{\alpha_1, \dots, \alpha_N\}$, N being the number of edges contained in Σ (under the hypotheses Σ turns out to be a trivalent graph), provides a local parametrization of the space of hyperbolic, resp. spherical, cone-structures near the given structure on M, and, in particular, there are no deformations that leave the cone-angles fixed. In the Euclidean case of the main theorem, the author proves that

$$H^1_{L^2}(M; \mathcal{E}_0) \simeq \{\omega \in \Omega(M, \mathcal{E}_0);\ \nabla\omega = 0\},$$

where $\mathcal{E}_0 \subset \mathcal{E}$ is the parallel sub-bundle of infinitesimal translations.

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57N65 Algebraic topology of manifolds
58D10 Spaces of embeddings and immersions

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