Mycielski, Jan


The author advocates adoption of an extension ST of ZFC obtained by adding new axioms that: (a) “Simplify set theory, inducing regularities without excluding any interesting objects”; (b) “Enrich its universe with interesting objects”. Each new axiom should “constitute a natural law of thought”.

Here are the new axioms proposed in this paper. (1) $V = \text{OD}$, that is, all sets are ordinal-definable. This yields the axioms of regularity and choice and implies that the definable elements of any model $M$ form an elementary submodel of $M$. (2) GCH, the Generalized Continuum Hypothesis. The author cites, in addition to the well-known simplifying advantages of GCH, other consequences that outweigh arguments against it. (3) SH, Suslin’s Hypothesis. In addition to the natural appeal of the usual form of SH, the author mentions the following equivalent proposition: If every chain and every antichain of a tree $T$ is countable, then $T$ is countable. (4) The relativization of the axiom of determinacy AD with respect to the class $L(\mathbb{R})$ of sets constructible on the basis of the set $\mathbb{R}$ of real numbers. This proposition simplifies the theory of projective sets and implies that all uncountable sets of reals have perfect subsets, all sets of reals are Lebesgue-measurable and have the Baire property. (5) SC, the large cardinal axiom that asserts that, for every cardinal $\kappa$, there is a strongly compact cardinal larger than $\kappa$. Some consequences of SC are sketched, but they are a bit too technical to be easily summarized here.

The author acknowledges that his system SC may inspire the objections that the new axioms: (a) are now considered open problems; (b) oversimplify set theory; (c) may, according to Platonists, be false. The author counters these objections in the following way. With respect to (a), he answers that it is known that none of the new axioms is a consequence of the others. The reviewer does not see why this is a conclusive answer. With respect to objection (b), the answer given is that “If we agree that ST does not appear to impose any bounds on the consistency strength of its possible extensions, then the fear that it oversimplifies set theory has no motivation. Thus I feel that (b) is not true (at least at the present time).” Moreover, the author asserts that recent theories proposed by Woodin are not attractive enough to convince us to give up GCH. With respect to (c), the author argues for a non-Platonist view of set theory, based, it appears, on a constructive interpretation of the quantifiers, but it is very unlikely that he can convert many mathematicians to such a radically new philosophy.

Reviewer: Elliott Mendelson (Flushing)

MSC:

03E30 Axiomatics of classical set theory and its fragments
03E65 Other set-theoretic hypotheses and axioms

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