Yan, Xiang-Ping; **Li, Wan-Tong**

Hopf bifurcation and global periodic solutions in a delayed predator – prey system. (English)

Zbl 1090.92052
Appl. Math. Comput. 177, No. 1, 427-445 (2006).

Summary: This paper is concerned with a delayed predator – prey system with same feedback delays of predator and prey species to their growth, respectively. We prove that a sequence of Hopf bifurcations occur at the positive equilibrium as the delay increases monotonously from zero. By using the theory of normal norm and center manifold reduction, an explicit algorithm for determining the direction of Hopf bifurcations and the stability of bifurcating periodic solutions is derived. In addition, the global existence results of periodic solutions bifurcating from Hopf bifurcations are established by using a global Hopf bifurcation result due to *J. Wu* [Symmetric functional differential equations and neural networks with memory. Trans. Am. Math. Soc. 350, No. 12, 4799–4838 (1998; Zbl 0905.34034)]. Finally, a numerical example supporting our theoretical predictions is also given.

Our findings are contrasted with recent studies on a delayed predator – prey system with the feedback time delay of prey species to its growth [*Y. Song* and *J. Wei*, Local Hopf bifurcation and global periodic solutions in a delayed predator-prey system. J. Math. Anal. Appl. 301, 1–21 (2005; Zbl 1067.34076)]. As the feedback time delay τ increases monotonously from zero, the positive equilibrium of the latter switches k times from stability to instability to stability. In contrast, the positive equilibrium of our system appears to lose the above property.

MSC:

92D40 Ecology
34K18 Bifurcation theory of functional-differential equations
34K20 Stability theory of functional-differential equations
92D25 Population dynamics (general)
37N25 Dynamical systems in biology
34K13 Periodic solutions to functional-differential equations

Keywords:

predator-prey system; time delay; Hopf bifurcation

Full Text: DOI

References:

[1] Chow, S.N.; Hale, J.K., Methods of bifurcation theory, (1982), Springer New York

[2] Faria, T., Stability and bifurcation for a delayed predator – prey model and the effect of diffusion, J. math. anal. appl., 254, 433-463, (2001) · Zbl 0973.35034

[3] Faria, T.; Magalhães, L.T., Normal form for retarded functional differential equations and applications to bogdanov – takens singularity, J. differential equations, 122, 201-224, (1995) · Zbl 0836.34069

[4] Hale, J.K., Theory of functional differential equations, (1977), Springer-Verlag New York · Zbl 0425.34048

[5] Hassard, B.D.; Kazarinoff, N.D.; Wan, Y.H., Theory and applications of Hopf bifurcation, (1981), Cambridge University Press Cambridge · Zbl 0474.34002

[6] He, X., Stability and delays in a predator – prey system, J. math. anal. appl., 198, 355-370, (1996) · Zbl 0873.34062

[7] Kuang, Y., Delay differential equations with application in population dynamics, (1993), Academic Press New York

[8] Liu, Z.; Yuan, R., Stability and bifurcation in a harmonic oscillator with delays, Chaos, solitons and fractals, 23, 551-562, (2005) · Zbl 1078.34050

[9] May, R.M., Time delay versus stability in population models with two and three trophic levels, Ecology, 4, 315-325, (1973)

[10] Song, Y.; Wei, J., Local Hopf bifurcation and global periodic solutions in a delayed predator – prey system, J. math. anal. appl., 301, 1-21, (2005) · Zbl 1067.34076

[11] Wang, W.; Ma, Z., Harmless delays for uniform persistence, J. math. anal. appl., 158, 256-268, (1991) · Zbl 0731.34085

[12] Wei, J.; Li, M., Global existence of periodic solutions in a tri-neuro network model with delays, Physica D, 198, 106-119, (2004) · Zbl 1062.34077

[13] Wei, J.; Ruan, S., Stability and bifurcation in a neural network model with two delays, Physica D, 130, 255-272, (1999) · Zbl 1066.34511

[14] Wu, J., Symmetric functional differential equations and neural networks with memory, Trans. amer. math. soc., 350, 4799-4838, (1998) · Zbl 0905.34034