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THE FIRST RESOURCE FOR MATHEMATICS

Mazza, Carlo; Voevodsky, Vladimir; Weibel, Charles

Lecture notes on motivic cohomology. (English) Zbl 1115.14010

Clay Mathematics Monographs 2. Providence, RI: American Mathematical Society (AMS); Cambridge, MA: Clay Mathematics Institute (ISBN 0-8218-3847-4/hbk). xiv, 216 p. (2006).

This book has been written by Carlo Mazza and Charles Weibel on the basis of the lectures on motivic cohomology that Vladimir Voevodsky gave at the Institute for Advenced Study in Princeton in 1999–2000. It is essentially self-contained and contains all the results on finite correspondences, presheaves with transfers, étale and Nisnevich sheaves with transfer, necessary to introduce the motivic complex $\mathbb{Z}(q)$ for every $q \geq 0$ and to prove the main properties of the motivic cohomology groups $H^{p,q}(X,\mathbb{Z})$. More generally, for any abelian group A, motivic cohomology with coefficients in A is a family of contravariant functors

$$H^{p,q}(-,A) : Sm/k \to \mathcal{A}b$$

from the category of smooth schemes over a field k to the category of abelian groups. In particular the book deals wit the following results for motivic cohomology (here X is a smooth scheme of finite type over a field)

$$H^{p,q}(X,A) = 0 \quad \text{for } q < 0; \tag{1}$$

$$H^{p,1}(X,\mathbb{Z}) \simeq \begin{cases} \mathcal{O}_X^* & \text{if } p = 1, \\ \operatorname{Pic} X & \text{if } p = 2, \\ 0 & \text{if } p \neq 1,2; \end{cases} \tag{2}$$

(3) for a field k : $H^{p,p}(\operatorname{Spec} k, A) = K_p^M(k) \otimes A$, where $K_*^M(k)$ is Milnor K-theory;

(4) for a strict Hensel local ring S over k and an integer n prime to the characteristic

$$H^{p,q}(S,\mathbb{Z}) \simeq \begin{cases} \mu_n^{\otimes q}(S) & \text{if } p = 0, \\ 0 & \text{if } p \neq 0, \end{cases}$$

where $\mu_n^{\otimes q}(S)$ is the group of n-roots of unit in S;

(5) one has $H^{p,q}(X,A) = \mathrm{CH}^q(X, 2q-p; A)$, where $\mathrm{CH}^i(X,j;A)$ denotes the Bloch higher Chow groups of X. In particular:

$$H^{2q,q}(X,A) = \mathrm{CH}^q(X) \otimes A,$$

where $\mathrm{CH}^q(X)$ is the classical Chow group of codimension q cycles modulo rational equivalence.

The book also contains the definitions and the main properties of the tensor triangulated category $DM_{\mathrm{Nis}}^{\mathrm{eff},-}(k,R)$ where R is a ring, as defined by Voevodsky and its subcategory of effective geometrical motives $DM_{\mathrm{gm}}^{\mathrm{eff}}$: homotopy, Mayer-Vietoris, projective bundle decomposition, blow-up triangles, Gysin sequence and the cancellation theorem.

In the case k is a perfect field which admits resolution of singularieties then Grothendieck's category of effective Chow motives $\mathcal{M}_{\mathrm{rat}}(k)$ embeds as a full subcategory into $DM_{\mathrm{gm}}^{\mathrm{eff}}(k,\mathbb{Z})$ and hence into $DM_{\mathrm{Nis}}^{\mathrm{eff},-}(k,\mathbb{Z})$.

Reviewer: Claudio Pedrini (Genova)

MSC:

14F42 Motivic cohomology; motivic homotopy theory
19E15 Algebraic cycles and motivic cohomology (K-theoretic aspects)
14C25 Algebraic cycles
14-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to algebraic geometry
14F20 Étale and other Grothendieck topologies and (co)homologies

Cited in **7** Reviews
Cited in **137** Documents