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On location and approximation of clusters of zeros of analytic functions. (English)

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Authors' abstract: At the beginning of the 1980s, M. Shub and S. Smale developed a quantitative analysis of Newton's method for multivariate analytic maps. In particular, their α -theory gives an effective criterion that ensures safe convergence to a simple isolated zero. This criterion requires only information concerning the map at the initial point of the iteration. Generalizing this theory to multiple zeros and clusters of zeros is still a challenging problem.

In this paper we focus on one complex variable function. We study general criteria for detecting clusters and analyze the convergence of Schröder's iteration to a cluster. In the case of a multiple root, it is well known that this convergence is quadratic. In the case of a cluster with positive diameter, the convergence is still quadratic provided the iteration is stopped sufficiently early. We propose a criterion for stopping this iteration at a distance from the cluster which is of the order of its diameter.

Reviewer: [Luigi Gatteschi \(Torino\)](#)

MSC:

- [65H05](#) Numerical computation of solutions to single equations
- [30B10](#) Power series (including lacunary series) in one complex variable
- [30C15](#) Zeros of polynomials, rational functions, and other analytic functions of one complex variable (e.g., zeros of functions with bounded Dirichlet integral)
- [65E05](#) General theory of numerical methods in complex analysis (potential theory, etc.)

Cited in **22** Documents

Keywords:

α -theory; Cluster approximations; Cluster location; Newton's operator; Pellet's criterion; Rouché's theorem; Schröder's operator

Software:

[MultRoot](#); [na20](#)

Full Text: [DOI](#)