

Bourbaki, Nicolas

Elements of mathematics. Algebra. Chapters 1–3. Reprint of the 1970 original. (Éléments de mathématique. Algèbre. Chapitres 1 à 3.) (French) [Zbl 1111.00001](#)
Berlin: Springer (ISBN 3-540-33849-7/pbk). xiii, 630 p. (2007).

N. Bourbaki’s imposing work “Éléments de Mathématique” is a monumental treatise of more than seven thousand pages, which must be seen as the perhaps most influential and sweeping mathematical opus of the 20th century. The first volume was published in 1939 and the most recent one in 1998, which amounts to a period of publishing activity of about sixty years, with varying thematic topics and different groups of (secret) authors. Through their fundamental treatise, the Bourbaki groups brought a totally new vision of pure mathematics, mainly by means of its profound reorganization, conceptual clarification, and terminological systematization. Until now, N. Bourbaki’s “Éléments de Mathématique” consists of ten books covering the foundations of some of the most important areas in modern pure mathematics. Book II of the entire treatise is devoted to the basics of the theory of algebraic structures, in its most abstract setting, and it is simply titled “Algebra”. Book II encompasses ten chapters, which were successively published between 1943 and 1980, originally in French.

Thanks to a very gratifying republishing initiative launched by Springer Verlag, all French originals of the volumes of “Éléments de Mathématique” have been made available again, this time as a complete collection of softcover editions at a reasonable price. Although there are excellent English (and Russian) translations of some of the (earlier) parts of Bourbaki’s work, the recent reproduction of the entire set exclusively refers to the French originals, apparently for the sake of historical accuracy and linguistic uniformity.

The volume under review is the faithful and unabridged reprinting of the first three chapters, of Bourbaki’s “Algèbre”, the English translations of which have been reviewed several times ([Zbl 0281.00006](#); [Zbl 0673.00001](#); [Zbl 0904.00001](#)), following the review of the, French original edition ([Zbl 0211.02401](#)) from 1970.

Thus, we may refer to these four reviews of the present book in order to avoid any superfluous repetition, or – even better – to Emil Artin’s unparalleled expert analysis of it given in 1953 (cf. *E. Artin* [The collected papers of Emil Artin. Edited by Serge Lang and John T. Tate. Reading, Mass. etc.: Addison-Wesley (1965; [Zbl 0146.00101](#))]). However, it should be stressed again that in particular Chapters 1–3 of Bourbaki’s “Algèbre” have been another historical milestone in regard of the development of modern abstract algebra from the structural, point of view, apart from B. B. van der Waerden’s epoch-making classic, text “Algebra I, II”, in that they set the current standards for organization, notation, and terminology of the subject. Also, it should be recollected that Chapters 1–3 of Bourbaki’s “Algèbre” systematically cover the following fundamental topics in abstract algebra:

Chapter 1 introduces general algebraic structures through their laws of composition, including magmas, monoids, groups, rings, fields, and their basic properties. Chapter 2 is devoted to linear algebra, which means here: modules, vector spaces, linear mappings, duality theory, matrices, determinants, tensor products, scalar base change, injective and projective limits, affine and projective spaces, rationality questions, and graded modules. Chapter 3 treats general algebras, examples of concrete algebras, graded algebras, tensor algebras, symmetric algebras, exterior algebras, norms and traces, derivations in algebras, coalgebras, and their geometric applications (p -vectors and Grassmannians). Now as before, there is this legendary ample supply of complementing exercises and historical notes adding up to the everlasting value of this particular volume.

Chapters 1–3 of Bourbaki’s “Algèbre” will persist as an indispensable source book of basic abstract algebra, also for future generations, with a companion in textbook-style given by the French standard primer “Cours d’Algèbre” of the former Bourbaki member, Roger Godement, first published in 1963.

Reviewer: [Werner Kleinert \(Berlin\)](#)

MSC:

00A05 Mathematics in general

Cited in **2** Reviews
Cited in **4** Documents

Keywords:

algebraic structures; groups; rings; fields; modules; algebras; linear algebra; multilinear algebra; affine geometry; projective geometry