Ceccherini-Silberstein, Tullio; **Scarabotti, Fabio**; **Tolli, Filippo**
Trees, wreath products and finite Gelfand pairs. (English) Zbl 1106.43006
Adv. Math. 206, No. 2, 503-537 (2006).

Let G be a finite group and K a subgroup of G. Let $L(X)$ denote the complex-valued functions on X. The pair (G, K) is a Gelfand pair if the algebra $L(K/G \setminus K)$ of bi-K-invariant functions is commutative.

Let T be a finite rooted tree of depth m and let $r = \{r_1, r_2, \ldots, r_m\}$ be an m-tuple of integers ≥ 2. T is of type r when each vertex at distance k from the root has exactly r_{k+1} sons, for $k = 0, 1, 2, \ldots, m-1$. If s is another m-tuple with $1 \leq s_k \leq r_k$ then $V(r, s)$ denotes the variety of subtrees of T of type s. Then $V(r, s) = \mathrm{Aut}(T)/K(r, s)$ where $K(r, s)$ is the stabilizer of a fixed T' in $V(r, s)$. The authors show that $(\mathrm{Aut}(T), K(r, s))$ is a Gelfand pair. This generalizes known examples: the ultrametric space, the Hamming scheme, and the Johnson scheme.

Reviewer: Benjamin B. Wells jun. (Charlottesville)

MSC:

43A90	Harmonic analysis and spherical functions	Cited in **2** Reviews
20B25	Finite automorphism groups of algebraic, geometric, or combinatorial structures	Cited in **17** Documents
20E08	Groups acting on trees	
20E22	Extensions, wreath products, and other compositions of groups	
22D15	Group algebras of locally compact groups	
33C80	Connections of hypergeometric functions with groups and algebras, and related topics	
43A85	Harmonic analysis on homogeneous spaces	

Keywords:

finite Gelfand pairs; wreath products; rooted trees; finite ultrametric space; Hamming scheme; Johnson scheme

Full Text: DOI

References:

[1] Akazawa, H.; Mizukawa, H., Orthogonal polynomials arising from the wreath products of a dihedral group with a symmetric group, J. combin. theory ser. A, 104, 2, 371-380, (2003) · Zbl 1064.33009

[2] Bailey, R.A., Association schemes, designed experiments, algebra and combinatorics, Cambridge stud. adv. math., vol. 84, (2004), Cambridge Univ. Press · Zbl 1051.05001

[3] Bailey, R.A.; Praeger, Ch.E.; Rowley, C.A.; Speed, T.P., Generalized wreath products of permutation groups, Proc. London math. soc. (3), 47, 1, 69-82, (1983) · Zbl 0496.20018

[4] Bannai, E., Orthogonal polynomials in coding theory and algebraic combinatorics, (), 25-53

[5] Bannai, E.; Ito, T., Algebraic combinatorics, (1984), Benjamin Menlo Park, CA · Zbl 0555.05019

[6] Bass, H.; Otero-Espinar, M.V.; Rockmore, D.; Tresser, Ch., Cyclic renormalization and automorphism groups of rooted trees, Lecture notes in math., vol. 1621, (1996), Springer Berlin

[7] Beaumont, R.; Peterson, R., Set transitive permutation groups, Canad. J. math., 7, 35-42, (1955) · Zbl 0064.02504

[8] Bekka, M.B.; de la Harpe, P., Irreducibility of unitary group representations and reproducing kernels Hilbert spaces, Expo. math., 21, 2, 115-149, (2003), Appendix by the authors in collaboration with Rostislav Grigorchuk · Zbl 1037.22009

[9] Bergeron, N.; Garsia, A., Zonal polynomials and domino tableaux, Discrete math., 88, 3-15, (1992) · Zbl 0759.05098

[10] Ceccherini-Silberstein, T.; Leonov, Yu.; Scarabotti, F.; Tolli, F., Generalized kaloujnine groups, uniseriality and height of automorphisms, Internat. J. algebra comput., 15, 3, 503-527, (2005) · Zbl 1151.20019

[11] T. Ceccherini-Silberstein, F. Scarabotti, F. Tolli, Finite Gelfand pairs and their applications to probability and statistics, J. Math. Sci. (New York), in press · Zbl 1173.43001

[12] Delsarte, Ph., An algebraic approach to the association schemes of coding theory, Philips res. rep. suppl., 10, (1973), Available

at: · Zbl 1075.05606

[13] Delsarte, Ph., Hahn polynomials, discrete harmonics and \textit{t}-designs, SIAM J. appl. math., 34, 154-166, (1978) · Zbl 0429.05031

[14] Diaconis, P., Group representations in probability and statistics, (1988), IMS Hayward, CA · Zbl 0695.60012

[15] Diaconis, P.; Shahshahani, M., Time to reach stationarity in the Bernoulli-Laplace diffusion model, SIAM J. math. anal., 18, 208-218, (1987) · Zbl 0617.60009

[16] Diaconis, P.; Graham, R.L.; Morrison, J.A., Asymptotic analysis of a random walk on a hypercube with many dimensions, Random structures algorithms, 1, 1, 51-72, (1990) · Zbl 0723.60085

[17] Dunkl, Ch.F., A krawtchouk polynomial addition theorem and wreath products of symmetric groups, Indiana univ. math. J., 25, 335-358, (1976) · Zbl 0326.33008

[18] Dunkl, Ch.F., An addition theorem for Hahn polynomials: the spherical functions, SIAM J. math. anal., 9, 627-637, (1978) · Zbl 0386.33008

[19] Dunkl, Ch.F., Orthogonal functions on some permutation groups, (), 129-147

[20] Dunkl, Ch.F.; Ramirez, D.E., Krawtchouk polynomials and the symmetrization of hypergroups, SIAM J. math. anal., 5, 351-366, (1974) · Zbl 0249.43006

[21] Durbin, J.R., Spherical functions on wreath products, J. pure appl. algebra, 10, 2, 127-133, (1977/1978) · Zbl 0369.43015

[22] Figà-Talamanca, A., An application of Gelfand pairs to a problem of diffusion in compact ultrametric spaces, (), 51-67 · Zbl 0985.60004

[23] A. Garsia, Gelfand pairs in finite groups, unpublished MIT manuscript, 1985

[24] Grigorchuk, R.I., Just infinite branch groups, (), 121-179 · Zbl 0982.20024

[25] Hanaki, A.; Hirotsuka, K., Irreducible representations of wreath products of association schemes, J. algebraic combin., 18, 1, 47-52, (2003) · Zbl 1021.05098

[26] Harary, F., Exponentiation of permutation groups, Amer. math. monthly, 66, 572-575, (1959) · Zbl 0089.01602

[27] Huppert, B., Character theory of finite groups, De gruyter expos. math., vol. 25, (1998), de Gruyter · Zbl 0932.20007

[28] James, G.D.; Kerber, A., The representation theory of the symmetric group, Encyclopedia math. appl., vol. 16, (1981), Addison Wesley Reading, MA

[29] Kerber, A.; Tappe, J., On permutation characters of wreath products, Discrete math., 15, 151-161, (1976) · Zbl 0335.20021

[30] Klimyk, A.U.; Vilenkin, N.Ja., Representation of Lie groups and special functions, vol. 2, Math. appl., vol. 81, (1993), Kluwer Academic Publishers Dordrecht · Zbl 0826.22001

[31] Letac, G., Problemes classiques de probabilité sur un couple de Gelfand, () · Zbl 0463.60010

[32] Letac, G., LES fonctions spheriques d'un couple de Gelfand symmetrique et LES chaines de Markov, Adv. in appl. probab., 14, 272-294, (1982) · Zbl 0482.60011

[33] Mizukawa, H., Zonal spherical functions on the complex reflection groups and $(n + 1, m + 1)$-hypergeometric functions, Adv. math., 184, 1-17, (2004) · Zbl 1054.33011

[34] Mizukawa, H.; Tanaka, H., $(n + 1, m + 1)$-hypergeometric functions associated to character algebras, Proc. amer. math. soc., (2004) · Zbl 1059.33020

[35] Saxl, J., On multiplicity-free permutation representations, (), 337-353 · Zbl 0578.20006

[36] Scarabotti, F., Fourier analysis of a class of finite Radon transforms, SIAM J. discrete math., 16, 4, 545-554, (2003) · Zbl 1041.44002

[37] Serre, J.P., Linear representations of finite groups, (1977), Springer New York

[38] Simon, B., Representations of finite and compact groups, (1996), Amer. Math. Soc.

[39] Stanley, R.P., Enumerative combinatorics, vol. 1, (1997), Cambridge Univ. Press · Zbl 0889.05001

[40] Stanton, D., Orthogonal polynomials and Chevalley groups, (), 87-128

[41] Stanton, D., Harmonics on posets, J. combin. theory ser. A, 40, 136-149, (1985) · Zbl 0573.06001

[42] Stanton, D., An introduction to group representations and orthogonal polynomials, (), 419-433

[43] Tarnanen, H.; Aaltonen, M.; Goethals, J.-M., On the nonbinary Johnson scheme, European J. combin., 6, 3, 279-285, (1985) · Zbl 0577.94014

[44] Terras, A., Fourier analysis on finite groups and applications, London math. soc. stud. texts, vol. 43, (1999), Cambridge Univ. Press · Zbl 0928.43001

[45] Vere-Jones, D., Finite bivariate distributions and semigroups of non-negative matrices, Quart. J. math. Oxford ser. (2), 22, 247-270, (1971) · Zbl 0215.40201

[46] Wielandt, H., Finite permutation groups, (1964), Academic Press New York · Zbl 0138.02501