

**Molnár, Lajos**

**Selected preserver problems on algebraic structures of linear operators and on function spaces.** (English) [Zbl 1119.47001](#)

*Lecture Notes in Mathematics* 1895. Berlin: Springer (ISBN 3-540-39944-5/pbk). xiv, 232 p. (2007).

The book under review is based on the author's dissertation for the highest Hungarian scientific degree and it contains mainly his own results. The idea of the monograph is an attempt to cover three mathematical sources where some transformations preserve (i) a quantity of elements, (ii) a distinguished set of elements, or (iii) a given relation among the elements of the underlying set. We often meet such problems in the study of linear operators in a Hilbert space. Such results are, for example, different types of Wigner's or Uhlhorn's theorem.

The book is divided into an introduction and three chapters with an appendix. The introduction describes the motivation of the book and outlines the survey and main results of each chapter. The first chapter deals with some linear and multiplicative preserver problems on operator algebras and function algebras. Here, the author focusses on describing linear transformations on operator algebras which preserve operators with infinite rank and infinite corank. Under some conditions, such transformations  $\phi$  are of the form  $\phi(X) = AXB$  or  $\phi(X)X^{tr}B$ , where  $A, B$  are some operators and  $^{tr}$  stands for the transpose. In addition, there is described the structure of linear maps on a von Neumann algebra preserving the extreme points of the unit ball, and diameter preserving linear bijections of  $C(X)$  are presented.

Chapter 2, Preservers on quantum structures, deals mainly with different types of the Wigner theorem and Uhlhorn-type results, like transformations of the set of  $n$ -dimensional subspaces of a Hilbert space preserving principal angles, and orthogonality preserving transformations on indefinite inner product spaces. The description of all bijective transformations of the set of all states preserving the fidelity, that is, the function  $F(A, B) := \text{tr}(A^{1/2}BA^{1/2})^{1/2}$ , where  $A, B$  are von Neumann operators. In addition, the chapter also contains some results concerning preservers on the set of Hermitian operators. We recall that an effect of a Hilbert space is any Hermitian operator between the zero and the identity. The author gives a description of sequential isomorphisms between the set of effects of a  $C^*$ -algebra.

The last chapter is dedicated to some properties of local automorphisms and local isometries. A typical result says that the automorphism and isometry groups of  $B(H)$  are topologically reflexive. Moreover, reflexivity of the automorphism and isometry groups of  $C(X)$  and reflexivity of the automorphism and isometry groups of the suspension of  $C(X)$  are described. The final results are about 2-local automorphisms of operator algebras on Banach spaces and of some quantum structures. The references contain 300 items.

The monograph under review collects many important and highly nontrivial results and efforts. It is important to recall that the basic material is based on the research done by the author, who belongs to the eminent researchers in this field. The style is very fresh, the author is keeping the regard of the reader permanently on his trip through the book. A typical feature of the author's method is that each subsection first describes the problem, formulates the results and, finally, presents the proofs. I recommend to book for students and experts interested in operator algebra, noncommutative measure theory and mathematical foundations of quantum physics. The monograph is welcome in the quantum structures realm.

Reviewer: [Anatolij Dvurečenskij \(Bratislava\)](#)

**MSC:**

- 47-02 Research exposition (monographs, survey articles) pertaining to operator theory
- 47B49 Transformers, preservers (linear operators on spaces of linear operators)
- 81R15 Operator algebra methods applied to problems in quantum theory

Cited in **1** Review  
Cited in **97** Documents

**Keywords:**

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