Zou, Jing

A sublinear algorithm for the recovery of signals with sparse Fourier transform when many samples are missing. (English) Zbl 1278.94024

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Summary: We present a sublinear randomized algorithm to compute a sparse Fourier transform for nonequispaced data of a special type. More precisely, we address the situation where a signal S is known to consist of N equispaced time samples, of which only $L < N$ samples are available. If the ratio $p = L/N$ is much smaller than 1, the available data typically look like nonequispaced samples, with little or no visible trace of the equispacing of the full set of N samples. We extend an approach for equispaced data that was presented in [the author et al., J. Comput. Phys. 211, No. 2, 572–595 (2006; Zbl 1085.65128)]; the extended algorithm reconstructs, from the incomplete data, a near-optimal B-term representation R with high probability $1 - \delta$, in time and space $\text{poly}(B, \log(N), \log(1/\delta), \varepsilon^{-1})$, such that

$$\|S - R\|_2^2 \leq (1 + \varepsilon)\|S - R_{\text{opt}}^B\|_2^2,$$

where R_{opt}^B is the optimal B-term Fourier representation of signal S. The sublinear $\text{poly}(\log N)$ time is compared to the superlinear $O(L1 + (d - 1)/\beta \log L)$ time requirement of the present best known inverse nonequispaced fast Fourier transform (INFFT) algorithms, in the sense of weighted norm with the number of dimensions d and smoothness parameter β. Numerical experiments support the advantage of our algorithm in speed over other methods for sparse signals: it already outperforms the INFFT for large but realistic size N and works well even in the situation of a large percentage of missing data and in the presence of large noise.

MSC:

94A12	Signal theory (characterization, reconstruction, filtering, etc.)
42A16	Fourier coefficients, Fourier series of functions with special properties, special Fourier series
68W20	Randomized algorithms
65T50	Numerical methods for discrete and fast Fourier transforms

Keywords:

sublinear randomized algorithm; sparse Fourier transform for nonequispaced data; optimal B-term Fourier representation of signal

Full Text: DOI

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