Braitt, Milton S.; **Silberger, Donald**
Subassociative groupoids. (English) Zbl 1123.20059
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Summary: When $\langle G;\diamond\rangle$ is a groupoid with binary operation $\diamond\colon G^2 \to G$, and when $k \in \mathbb{N} := \{1, 2, 3, \dots, \}$, then $F^\sigma(k)$ denotes the set of all formal products $\mathbf{u}$ on k independent variables. It is well known that $|F^\sigma(k)| = C(k)$, where $C(k)$ is the k-th Catalan number.

Each word $\mathbf{u} \in F^\sigma(k)$ induces a function $\mathbf{u}\colon G^k \to G$ given by $\mathbf{u}\colon \vec{g} \mapsto \mathbf{u}(\diamond, \vec{g})$, where $\mathbf{u}(\diamond, \vec{g})$ is the interpretation in $\langle G, \diamond\rangle$ of $\mathbf{u}$ as a $\diamond$-product of the sequence $\vec{g} := \langle g_0, g_1, \dots, g_{k-1}\rangle \in G^k$.

Write $\mathbf{u} =_\diamond \mathbf{v}$ for $\{\mathbf{u}, \mathbf{v}\} \subseteq F^\sigma(k)$ iff $\mathbf{u}(\diamond, \vec{g}) = \mathbf{v}(\diamond, \vec{g})$ whenever $\vec{g} \in G^k$. This $=_\diamond$ is an equivalence relation on the set $F^\sigma := \bigcup\{F^\sigma(k) : k \in \mathbb{N}\}$. The sequence $\mathbf{SaT}(\langle G;\diamond\rangle) := \langle |F^\sigma(k)/ =_\diamond| \rangle_{k=2}^{\infty}$ presents the subassociativity types of $\langle G;\diamond\rangle$.

We calculate $\mathbf{SaT}(G)$ for a few evocative groupoids $G := \langle G;\diamond\rangle$, and we initiate a study of the partitions $F^\sigma(k)/ =_\diamond$. Each equivalence class of the completely free groupoid F^σ is a singleton, and so F^σ realizes the theoretical minimum k-associativity for each $k \in \mathbb{N}$. We propose for each k a minimally k-associative class of finite groupoids.

MSC:

20N02 Sets with a single binary operation (groupoids)
08A02 Relational systems, laws of composition
05A15 Exact enumeration problems, generating functions

Keywords:

Catalan numbers; equivalences; subassociativity types; free groupoids; finite groupoids