Kaur, Kulwinder

Integrability and L^1-convergence of Rees-Stanojević sums with generalized semi-convex coefficients of non-integral orders. (English) Zbl 1111.42001

Arch. Math., Brno 41, No. 4, 423-437 (2005).

Let $g(x)=\sum_{k=1}^{\infty} a_k \cos kx$ be a Fourier cosine series and $g_n(x)=\frac{1}{2}\sum_{k=0}^{n}\Delta a_k+\sum_{k=1}^{n}\sum_{j=k}^{n}(\Delta a_j)\cos kx$. The main result of the paper reads as follows:

If $a_n\to 0$ as $n\to\infty$ and $\sum_{n=1}^{\infty} n^{\alpha}\left|\Delta^{\alpha+1}a_{n-1}+\Delta^{\alpha+1}a_n\right|<\infty$, where $\alpha>0$, then $g_n(x)$ converges in L^1-metric to $g(x)$ if and only if $\Delta a_n \log n=o(1)$ as $n\to\infty$. This statement is the extension of a similar result presented in *K. Kaur* and *S. S. Bhatia* [Int. J. Math. Math. Sci. 30, 645–650 (2002; Zbl 1010.42017)], where it is supposed that α is an integer. Recall that the noninteger difference $\Delta^{\alpha}a_n$ is defined by the formula $\Delta^{\alpha}a_n=\sum_{m=0}^{\infty}A_m^{-\alpha-1}a_{n+m}$, where A's are binomial coefficient in the expansion $(1-x)^{-\alpha-1}=\sum_{k=0}^{\infty}A_k^{\alpha}x^k$.

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MSC:

42A20 Convergence and absolute convergence of Fourier and trigonometric series
42A32 Trigonometric series of special types (positive coefficients, monotonic coefficients, etc.)

Keywords:

conjugate Cesàro mean; Fourier cosine series

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