Dajić, Alegra; **Koliha, J. J.**

Positive solutions to the equations $AX = C$ and $XB = D$ for Hilbert space operators. (English)

Zbl 1120.47009
J. Math. Anal. Appl. 333, No. 2, 567-576 (2007).

C. G. Khatri and *S. K. Mitra* [SIAM J. Appl. Math. 31, 579–585 (1976; Zbl 0359.65033)] studied positive and general solutions of the matrix equations $AX = C$ and $XB = D$ and $AXB = C$. *S. V. Phadke* and *N. K. Thakare* [Linear Algebra Appl. 23, 191–199 (1979; Zbl 0403.47005)] attempted to describe the Hermitian and positive solutions for Hilbert space operators.

In the paper under review, the authors find some conditions for the existence of Hermitian and positive solutions of the equations $AX = C$ and $XB = D$, where A, B, C, D are bounded linear operators between Hilbert spaces. They show that if A and CA^* have closed ranges, then $AX = C$ has a positive solution X if and only if $CA^* \geq 0$ and the range of C is contained in the range of CA^*. In fact, the general positive solution is given by $X = C^*(CA^*)^- C + (I - A^- A)S(I - A^- A)^*$, where S is positive and $(CA^*)^-$ and A^- are arbitrary inner inverses of CA^* and A, respectively. (Recall that A has an inner inverse A^- if $AA^- A = A$.)

Reviewer: Maryam Amyari (Mashhad)

MSC:

47A62	Equations involving linear operators, with operator unknowns	Cited in **1** Review
47A05	General (adjoints, conjugates, products, inverses, domains, ranges, etc.)	Cited in **34** Documents
15A24	Matrix equations and identities	

Keywords:

Hilbert space; operator equation; positive solution; common positive solution

Full Text: DOI

References:

[1] Albert, A., Conditions for positive and nonnegative definiteness in terms of pseudoinverses, SIAM J. appl. math., 17, 434-440, (1969) · Zbl 0265.15002

[2] Ben-Israel, Adi; Greville, T.N.R., Generalized inverses, theory and applications, CMS book math., vol. 15, (2003), Springer-Verlag New York · Zbl 1026.15004

[3] Campbell, S.L.; Meyer, C.D., Generalized inverses of linear transformations, (1979), Pitman London · Zbl 0417.15002

[4] Cvetković-Ilić, D.S.; Djordjević, D.S.; Rakočević, V., Schur complements in $C^\ast$-algebras, Math. nachr., 278, 1-7, (2005)

[5] Khatri, C.G.; Mitra, S.K., Hermitian and nonnegative definite solutions of linear matrix equations, SIAM J. appl. math., 31, 579-585, (1976) · Zbl 0359.65033

[6] Patrício, P.; Puystjens, R., About the von Neumann regularity of triangular block matrices, Linear algebra appl., 332-334, 485-502, (2001) · Zbl 1002.15001

[7] Penrose, R., A generalized inverse for matrices, Proc. Cambridge philos. soc., 51, 406-413, (1955) · Zbl 0065.24603

[8] Phadke, S.V.; Thakare, N.K., Generalized inverses and operator equations, Linear algebra appl., 23, 191-199, (1979) · Zbl 0403.47005