

Jenei, Sandor

On the geometry of associativity. (English) Zbl 1131.20047
Semigroup Forum 74, No. 3, 439-466 (2007).

In this paper a geometric characterization of residuated semigroups is presented. But the aim is to develop a method for deciding associativity from the three-dimensional graph of an operation. To do this a geometric characterization of associativity is presented in the paper. This provides a deeper understanding of associativity, which turns out to be fruitful in conjecturing and proving algebraic results in the field of residuated lattices, and in establishing results in corresponding non classical logics. On the other hand, this geometric description has provided the intuition for a contribution to solving a long-standing open problem in the field of associative functions [see *S. Jenei*, Aequationes Math. 72, No. 1-2, 47-59 (2006; [Zbl 1101.39010](#))].

Throughout the paper the author illustrates his geometric characterization by computer generated pictures of surfaces in the unit cube. The aim of this paper is not only to introduce a kind of visualization aspect but, in addition, to understand algebraic relationships in a geometric manner. It is interesting to notice that when conjecturing algebraic results based on geometric motivations, one can first heuristically prove them using geometry. Then the steps of the geometric hint can be translated step by step into an algebraic proof.

Reviewer: [C. Pereira da Silva \(Curitiba\)](#)

MSC:

- [20M14](#) Commutative semigroups
- [06F05](#) Ordered semigroups and monoids
- [20N02](#) Sets with a single binary operation (groupoids)
- [03B52](#) Fuzzy logic; logic of vagueness
- [03E72](#) Theory of fuzzy sets, etc.

Cited in **9** Documents

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[groupoids](#); [residuated lattices](#); [rotation-invariance](#); [associativity](#)

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