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Relation between the Farrell-Jones conjectures in algebraic and Hermitian K -theory. (Relation entre les conjectures de Farrell-Jones en K -théories algébrique et hermitienne.) (French.

English summary) [[Zbl 1126.19005](#)]

Ann. Inst. Fourier 57, No. 1, 197-207 (2007).

Let A be a ring and let G be a discrete group. For each integer n , $J.-L. Loday$ defined in [*" K -théorie algébrique et représentations de groupes"*, *Ann. Sci. Éc. Norm. Supér.* (4) 9, 309–377 (1976; [Zbl 0362.18014](#))] an assembly map

$$\lambda_n : h_n(BG, \mathcal{K}_A) \rightarrow K_n(AG)$$

between the homology groups $h_n(BG, \mathcal{K}_A) = \pi_n(BG_+ \wedge \mathcal{K}_A)$ of BG with values in the algebraic K -theory spectrum \mathcal{K}_A of A and the K -theory groups of the group ring AG . In the case $A = \mathbb{Z}$ the morphisms λ_n have been conjectured to be isomorphisms for all n by $F. T. Farrell$ and $L. E. Jones$ [*"Isomorphism conjectures in algebraic K -theorie"*, *J. Am. Math. Soc.* 6, No. 2, 249–297 (1993; [Zbl 0798.57018](#))].

In a similar manner, for a ring A with involution $\bar{}$ and $1/2 \in A$, one can define assembly maps

$$\alpha_n : h_n(BG, \mathcal{L}_A) \rightarrow {}_\epsilon L_n(AG)$$

by replacing the K -theory spectrum \mathcal{K}_A of A by the ϵ -Hermitian K -theory spectrum \mathcal{L}_A of A and the K -groups $K_n(AG)$ by the corresponding ϵ -Hermitian K -groups ${}_\epsilon L_n(AG)$. Here ϵ is a central element in A satisfying $\epsilon\bar{\epsilon} = 1$. Again Farrell and Jones conjectured that the morphisms α_n are isomorphisms in the case that $A = \mathbb{Z}[\frac{1}{2}]$.

Using results of $M. Karoubi$'s [*"Le théorème fondamental de la K -théorie hermitienne"*, *Ann. Math.* (2) 112, 259–282 (1980; [Zbl 0483.18008](#))], the author shows the following: If the Farrell-Jones Conjecture holds in algebraic K -theory, then the validity of the Farrell-Jones Conjecture in Hermitian K -theory is equivalent to the fact that for some integer n the maps α_n and α_{n-1} are isomorphisms.

Reviewer: [Manfred Kolster \(Hamilton/Ontario\)](#)

MSC:

[19G38](#) Hermitian K -theory, relations with K -theory of rings

Keywords:

algebraic K -theory; Hermitian K -theory; assembly map; Farrell-Jones conjectures

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References:

- [1] Bartels, A.; Reich, H., On the Farrell-Jones conjectures for higher algebraic (K) -theory, (2003) · [Zbl 1073.19002](#)
- [2] Farrell, F. T.; Jones, L., Isomorphism conjectures in algebraic (K) -theory, *J. Amer. Math. Soc.*, 6, 2, 249-297, (1993) · [Zbl 0798.57018](#)
- [3] H. Bass, A. Heller; Swan, R. G., The Whitehead group of polynomial extension, *Inst. hautes études sci.*, 22, 61-79, (1964) · [Zbl 0248.18026](#)
- [4] Hsiang, W. C., Borel's conjecture, Novikov's conjecture and the (K) -theoretic analogue, (1989), World scientific book, Singapour · [Zbl 0744.57018](#)
- [5] Karoubi, M., Le théorème fondamental de la (K) -théorie hermitienne, *Annals of mathematics*, 112, 259-282, (1980) · [Zbl 0483.18008](#)
- [6] Loday, J.-L., (K) -théorie algébrique et représentation de groupes, *Ann. Sci. Ecole Normale Sup. Sér.* 4, 9, 3, 309-377, (1976) · [Zbl 0362.18014](#)
- [7] Whitehead, G. W., Generalised homology theories, *Trans. A. M. S.*, 102, 227-283, (1962) · [Zbl 0124.38302](#)

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