Schulze, Felix

Convexity estimates for flows by powers of the mean curvature. (English) Zbl 1150.53024

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The study of the mean curvature flow (or, in general, of one of its powers) have attracted many mathematicians during the last 30 years. For example, *G. Huisken* showed [J. Differ. Geom. 20, 237–266 (1984; Zbl 0556.53001)] that convex surfaces remain convex under that flow and contract to a ̀round´point in finite time.

In the paper under review, the author continues the study initiated in [Math. Z. 251, No. 4, 721–733 (2005; Zbl 1087.53062)] where he investigated the evolution of a closed convex hypersurface in $\mathbb{R}^{n+1}$, $F_0 : M^n \to \mathbb{R}^{n+1}$, with positive mean curvature H. There he showed that there exists a unique smooth solution $F(\cdot,t) : M^n \times [0,T) \to \mathbb{R}^{n+1}$ to the initial value problem given by the H^k-flow on a maximal finite interval $[0,T)$:

$$F(\cdot,0) = F_0(\cdot)$$
$$\frac{dF}{dt}(\cdot,t) = -H^k(\cdot,t)\nu(\cdot,t)$$

where $k > 0$ and ν is the outer unit normal such that $-H\nu = \mathbf{H}$ is the mean curvature vector.

In this paper the author presents an extension of the previous result by using that by the arithmetic-geometric mean inequality we have $0 \leq K/H^n \leq 1/n^n$, with equality on the right side if and only if all eigenvalues are equal. The main result of the paper states as follows.

"For $k \geq 1$ there exists a nonnegative constant $C(n,k) < 1/n^n$ such that the following holds: If the initial hypersurface is pinched in the sense that

$$\frac{K(p)}{H^n(p)} > C(n,k),$$

for all $p \in M$, then this is preserved under the H^k-flow. The constant $C(n,k)$ is increasing in k, $\lim_{k\to 1} C(n,k) = 0$ and $\lim_{k\to\infty} C(n,k) = 1/n^n$. Furthermore, the rescaled embeddings

$$\tilde{F}(\tau,p) := ((k+1)n^k(T-t))^{-1/(k+1)}(F(\tau,p) - x_0)$$

converge for $\tau \to \infty$ exponentially in the C^∞-topology to the unit sphere. Here $\tau := -(k+1)^{-1}n^{-k}\log(1-t/T)$, where T is the maximal time of existence of the unrescaled flow and x_0 is the point in $\mathbb{R}^{n+1}$ where the surfaces contract to".

The paper also contains an appendix, written jointly with O. Schnürer, where they give an extension for surfaces in the 3-dimensional Euclidean space. They show that for $1 \leq k \leq 5$ no initial pinching condition is needed to ensure that the rescaled embeddings converge to the unit sphere.

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MSC:

53C44 Geometric evolution equations (mean curvature flow, Ricci flow, etc.) (MSC2010)
35B40 Asymptotic behavior of solutions to PDEs

Full Text: EuDML

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