Host, Bernard

Arithmetic progressions in prime numbers (after B. Green and T. Tao). (Progressions arithmétiques dans les nombres premiers [d'après B. Green et T. Tao].) (French) Zbl 1175.11052

Séminaire Bourbaki. Volume 2004/2005. Exposés 938–951. Paris: Société Mathématique de France (ISBN 978-2-85629-224-2/pbk). Astérisque 307, 229-246, Exp. No. 944 (2006).

In [Acta Arith. 27, 199–245 (1975; Zbl 0303.10056)], *E. Szemerédi* proved that every set of integers of positive upper density contains arbitrarily long arithmetic progressions. Three decades later in [Ann. Math. (2), 167, No. 2, 481–547 (2008; Zbl 1191.11025); preprint http://arxiv1.library.cornell.edu/abs/math/0404188], *B. Green* and *T. Tao* proved the spectacular result that the sequence of primes contains arbitrarily long arithmetic progressions.

The current article gives a clear exposition in some detail of the ideas, key steps and the main hurdles to be overcome in this proof. The first ingredient is the Green-Tao-Szemerédi theorem, an extension of Szemerédi's theorem, which is stated in terms of a "pseudo-random measure". Secondly a suitable pseudo-random measure is established that can be applied to the prime numbers using sieve theory results due to *D. Goldston* and *C. Y. Yildirim* [preprint; http://front.math.ucdavis.edu/math.NT/0504336]. The author shows how these components combine to establish the result of Green and Tao.

For the entire collection see [Zbl 1105.00003].

Reviewer: Eira J. Scourfield (Egham)

MSC:

11N13	Primes in congruence classes	Cited in **3** Documents
11B25	Arithmetic progressions	
11A41	Primes	
37A45	Relations of ergodic theory with number theory and harmonic analysis (MSC2010)	

Keywords:

primes in arithmetic progression; theorem of Green and Tao; Szemerédi's theorem