

**Ahlgren, Scott; Barcau, Mugurel**

**Congruences for modular forms of weights two and four.** (English) Zbl 1144.11036  
J. Number Theory 126, No. 2, 193-199 (2007).

*F. Calegari* and *W. Stein* [in: Algorithmic number theory. 6th international symposium, ANTS-VI, Burlington, VT, USA, June 13–18, 2004. Proceedings. Berlin: Springer. Lecture Notes in Computer Science 3076, 140–152 (2004; [Zbl 1125.11320](#))] stated several conjectures about discriminants of Hecke algebras of prime level. In the paper under review, the authors prove the following one:

Let  $p$  be a prime,  $p \geq 5$ , and  $\Theta$  the Ramanujan operator. Let  $\mathcal{P}$  be the maximal ideal of  $\overline{\mathbb{Z}}_p$ . Suppose that  $f \in S_2(\Gamma_0(p), \overline{\mathbb{Z}}_p)$  and  $g \in S_4(\Gamma_0(p), \overline{\mathbb{Z}}_p)$  are Hecke eigenforms. Assume

$$\Theta f \equiv g \pmod{\mathcal{P}}.$$

Then the Atkin-Lehner eigenvalues of  $f$  and  $g$  have opposite signs.

They also mention another proof indicated by F. Calegari based on a theorem of *C. Breuil* and *A. Mézard* [Duke Math. J. 115, No. 2, 205–310 (2002; [Zbl 1042.11030](#))]. The proof given in the paper under consideration employs elementary, but very tricky arguments based on special properties of  $\Theta$ , the quasi modular form  $E_2$ , Eisenstein series of weight 2, and Serre's and Swinnerton-Dyer's theory of modular forms modulo  $p$ .

Reviewer: [Bernhard Heim \(Bonn\)](#)

#### MSC:

[11F33](#) Congruences for modular and  $p$ -adic modular forms  
[11F11](#) Holomorphic modular forms of integral weight

Cited in **1** Review  
Cited in **3** Documents

#### Keywords:

modular forms mod  $p$ ; congruences; Atkin-Lehner operator

**Full Text:** [DOI](#)

#### References:

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