Krichever, Igor M.

Integrable linear equations and the Riemann-Schottky problem. (English) Zbl 1132.14032

Ginzburg, Victor (ed.), Algebraic geometry and number theory. In Honor of Vladimir Drinfeld's 50th birthday. Basel: Birkhäuser (ISBN 978-0-8176-4471-0/hbk). Progress in Mathematics 253, 497-514 (2006).

The Riemann-Schottky problem consists in describing the locus formed by the Jacobian varieties of compact Riemann surfaces in the moduli space of principally polarized abelian varieties.

A new approach to this problem was proposed by S. P. Novikov in the middle of 1970s and it was based on the Krichever construction of theta-functional solutions for the Kadomstev-Petviashvili (KP) equation from an arbitrary compact Riemann surface (without boundary) with a marked point and some additional data. Such a solution takes a form $u(x,y,t) = -2\partial_x^2 \log\theta(Ux + Vy + Wt + Z)$ and Novikov conjectured that if there exists vectors U, V, and W such that this formula gives a solution of the KP equation and the abelian variety corresponding to this theta function is irreducible then this abelian variety is the Jacobian variety of some Riemann surface. After some substantial partial results of Dubrovin, Arbarello-de Concini, and Mulase, this conjecture was proved by *T. Shiota* [Invent. Math. 83, 333–382 (1986; Zbl 0621.35097)] and that gives the first solution to the Riemann-Schottky problem.

In the present paper the author proposes another characterization of Jacobians again based on the soliton theory. The KP equation is the compatibility condition for the system $(\partial_y - \partial_x^2 + u)\psi = 0$ and $(\partial_t - \partial_x^3 + \frac{3}{2}\partial_x + w)\psi = 0$. It is proved that for characterization of Jacobians it is enough to use only the first of this equation, i.e. an irreducible principally polarized abelian variety is the Jacobian if and only if there are vectors $U \neq 0, V$ and A such that the functions $u = -2\partial_x^2 \log\theta(Ux + Vy + Z)$ and $\psi = \frac{\theta(A+Ux+Vy+Z)}{\theta(Ux+Vy+Z)} e^{px+Ey}$ satisfy the equation $(\partial_y - \partial_x^2 + u)\psi = 0$. In its strongest form the results of the paper read that the Jacobians are characterized by some formal infinite-dimensional Calogero-Moser system. Recently the technique developed by the author allows him to characterize some other classes of abelian varieties in terms of soliton theory.

For the entire collection see [Zbl 1113.00007].

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MSC:

14H70	Relationships between algebraic curves and integrable systems
14H40	Jacobians, Prym varieties
14K05	Algebraic theory of abelian varieties
37K20	Relations of infinite-dimensional Hamiltonian and Lagrangian dynamical systems with algebraic geometry, complex analysis, and special functions
14H42	Theta functions and curves; Schottky problem

Keywords:

abelian variety; Riemann-Schottky problem; Calogero-Moser system; Kadomtsev-Petviashvili equation