Cégielski, Patrick; Richard, Denis; Vsemirnov, Maxim

On the additive theory of prime numbers. (English) Zbl 1153.11061

Fundam. Inform. 81, No. 1-3, 83-96 (2007).

In connection to the open problem of "the decidability or undecidability of $\mathrm{Th}(\mathbb{N},+,\mathbb{P})$ where $\mathbb{P}$ is the set of primes" the paper considers the problem of the decidability of $\mathrm{Th}(\mathbb{N},+,n\mapsto p_n)$ where p_n denotes the $(n+1)$-th prime number. The latter problem and the problem of whether p_n's are definable in $(\mathbb{N},+,\mathbb{P})$ are still open.

Since, by the prime numbers theorem, p_n can be approximated by $n\log(n)$, the author studies $\mathrm{Th}(\mathbb{N},+,n\mapsto n\log(n))$ and $\mathrm{Th}(\mathbb{N},+,n\mapsto n\log_2(n))$ and proves their undecidability. Here $\log(n)$ and $\log_2(n)$ are the integer part (floor) of Neperian logarithm and binary logarithm of n. In the second part of the paper, it is proved that multiplication is definable in $(\mathbb{N},+,n\mapsto p_n,n\mapsto r_n)$, where r_n is the remainder of p_n divided by n, by an existential formula. Thus $\mathrm{Th}_\exists(\mathbb{N},+,n\mapsto p_n,n\mapsto r_n)$ is undecidable.

Reviewer: Saeed Salehi (Tabriz)

MSC:

11U05 Decidability (number-theoretic aspects)
03B25 Decidability of theories and sets of sentences

Full Text: arXiv