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Semi-classical analysis for nonlinear Schrödinger equations. (English) Zbl 1153.35070
Hackensack, NJ: World Scientific (ISBN 978-981-279-312-6/hbk). xi, 243 p. (2008).

This book presents a consistent account of mathematically rigorous and formal analysis of the evolution of wave fields obeying multidimensional nonlinear Schrödinger (NLS) equations with an external potential in the multidimensional space, and with the self-focusing nonlinear term with an arbitrary power. In particular, such equations with the cubic nonlinearity represent the Gross-Pitaevskii equations, i.e., fundamental dynamical equations governing the evolution of the mean-field wave function in Bose-Einstein condensates. The NLS equations with the cubic nonlinearity are also fundamentally important models in nonlinear optics. The book is focused on the “quasi-classical” limit case (so called in terms of quantum mechanics), when the Laplacian in the equation appears with a small parameter in front of it, so that this term tends to be small in comparison with the nonlinear and potential ones. The book is composed of two parts, each being essentially self-contained.

In the first part, the analysis is based on a technique which is an extension of the WKB approximation, the latter being a well-known ingredient of quantum mechanics. The analysis of the WKB approximation is first performed for the linear case, and then extended for the case of the full nonlinear equation. The second part of the book is dealing with the critically important case when the WKB approximation breaks down, and singularities in the form of *caustics* appear. In particular, special attention is paid to the most difficult case of the so-called supercritical NLS equation, defined by condition $D(p - 1) > 4$, where D is the spatial dimension, and p the nonlinearity power.

Reviewer: [Boris A. Malomed \(Tel Aviv\)](#)

MSC:

- 35Q55** NLS equations (nonlinear Schrödinger equations)
- 35-02** Research exposition (monographs, survey articles) pertaining to partial differential equations
- 81Q20** Semiclassical techniques, including WKB and Maslov methods applied to problems in quantum theory

Cited in 1 Review Cited in 26 Documents
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Keywords:

[WKB approximation](#); [Gross-Pitaevskii equation](#); [caustics](#)