Benjamini, Itai; Sznitman, Alain-Sol
Giant component and vacant set for random walk on a discrete torus. (English) Zbl 1141.60057
J. Eur. Math. Soc. (JEMS) 10, No. 1, 133-172 (2008).

The authors consider symmetric nearest neighbour random walk X on the d-dimensional ($d \geq 3$) integer lattice torus $E := \big(\mathbb{Z}/(N\mathbb{Z})\big)^d$ of side-length N. It is well-known that the cover time is of order $N^d \log N$ if $d \geq 3$. In this paper, Benjamini and Sznitman investigate the percolative structure of the set $V \subset E$ of sites that are *not* visited by X up to time uN^d, where $u > 0$ is assumed to be small. First of all they show (Corollary 4.5) that there are constants $c = c(d)$ and $c' = c'(d)$ such that

$$\lim_{N\to\infty} \mathbb{P}[e^{-cu} \leq \#V/N^d \leq e^{-c'u}] = 1.$$

In Theorem 1.2 it is shown for $d \geq 4$ and for any $\beta \in (0,1)$ and $K > 0$ that if $u > 0$ is small enough, then with probability tending to one (as $N \to \infty$), with probability tending to one, every point $x \in E$ is in distance at most N^β to some point in V that is in a straight line segment in V of length at least $K \log N$. The next results hold for d larger than some d_0 (and which the reviewer computed to be actually $d_0 = 123$). In Corollary 2.6 it is shown for $d \geq d_0$ that if $u > 0$ is small enough, then with probability tending to one (as $N \to \infty$) there is a unique connected component $O \subset V$ that contains straight line segments (in any of the d directions) of size $c_0 \log N$ (where c_0 is a dimension dependent constant). Moreover, in Corollary 4.6 it is shown that for $d \geq d_0$, for any $\gamma \in (0,1)$ and $u = u(\gamma) > 0$ sufficiently small, with probability tending to one, the cardinality of O is a least γN^d. That is, O contains a substantial fraction (depending on u) of points of E. However, it remains open if V contains more connected components of substantial size.

Reviewer: Achim Klenke (Mainz)

MSC:

60K35 Interacting random processes; statistical mechanics type models; percolation theory
60G50 Sums of independent random variables; random walks
82B41 Random walks, random surfaces, lattice animals, etc. in equilibrium statistical mechanics

Keywords:
probability theory; random walk; percolation; stochastic processes; coupon collector

Full Text: DOI arXiv

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