Butler, L. T.; **Levit, B.**

A Bayesian approach to the estimation of maps between Riemannian manifolds. (English)

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Math. Methods Stat. 16, No. 4, 281-297 (2007).

Summary: Let Θ be a smooth compact oriented manifold without boundary, imbedded in a Euclidean space E^s, and let γ be a smooth map of Θ into a Riemannian manifold Λ. An unknown state $\theta \in \Theta$ is observed via $X = \theta + \varepsilon\xi$, where $\varepsilon > 0$ is a small parameter and ξ is a white Gaussian noise. For a given smooth prior λ on Θ and smooth estimators $g(X)$ of the map γ we derive a second-order asymptotic expansion for the related Bayesian risk. The calculation involves the geometry of the underlying spaces Θ and Λ, in particular, the integration-by-parts formula. Using this result, a second-order minimax estimator of γ is found based on the modern theory of harmonic maps and hypoelliptic differential operators.

MSC:

62C10 Bayesian problems; characterization of Bayes procedures
62C20 Minimax procedures in statistical decision theory
62F12 Asymptotic properties of parametric estimators
53B20 Local Riemannian geometry
53C17 Sub-Riemannian geometry

Keywords:

Bayes estimators; minimax estimators; sub-Riemannian geometry; sub-Laplacian; harmonic maps

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