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Strong A_∞-weights and scaling invariant Besov capacities. (English) Zbl 1149.46028

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This article studies strong A_∞-weights and Besov capacities as well as their relationship to Hausdorff measures. A weight ω is said to be an A_∞-weight if there exist constants $C \geq 1$ and $q > 1$ such that $\left(\frac{1}{|B|}\int_B \omega(x)^q\,dx\right)^{1/q} \leq \frac{1}{|B|}\int_B \omega(x)\,dx$ for all balls $B \subset \mathbb{R}^n$. An A_∞-weight ω is called a strong A_∞-weight if there exists a distance function δ^1_μ on $\mathbb{R}^n$ and a positive constant C such that $C^{-1}\delta_\mu(x,y) \leq \delta^1_\mu(x,y) \leq C\delta_\mu(x,y)$, where μ is the measure on $\mathbb{R}^n$ with density ω and $\delta_\mu(x,y) = \mu(B_{x,y})^{1/n}$, where $B_{x,y}$ denotes the smallest closed ball which contains the points x and y.

The author proves that in the Euclidean space $\mathbb{R}^n$ with $n \geq 2$, whenever $n-1 < s \leq n$, a function u yields a strong A_∞-weight of the form $\omega = e^{nu}$ if the distributional gradient ∇u has sufficiently small $\|\cdot\|_{\mathcal{L}^{n,n-s}(\mathbb{R}^n;\mathbb{R}^n)}$-norm, where $\mathcal{L}^{n,n-s}(\mathbb{R}^n)$ denotes the Morrey space of vector-valued measurable functions and

$$\|\nabla u\|_{\mathcal{L}^{n,n-s}(\mathbb{R}^n;\mathbb{R}^n)} = \sup_{x\in\mathbb{R}^n}\sup_{r>0}\left(r^{-(n-s)}\int_{B(x,r)} |\nabla u(y)|^n\,dy\right)^{1/n}.$$

As a corollary of this result, the author obtains strong A_∞-weights of the form $\omega = e^{nu}$, where u is a distributional solution of $-\mathrm{div}(|\nabla u|^{n-2}\nabla u) = \mu$ whenever μ is a signed Radon measure with small total variation. Similarly, the author also proves that if $2 \leq n < p < \infty$, then $\omega = e^{nu}$ is a strong A_∞-weight whenever the Besov B_p-seminorm $[u]_{B_p(\mathbb{R}^n)}$ of u is sufficiently small, where $[u]_{B_p(\mathbb{R}^n)} = (\int_{\mathbb{R}^n}\int_{\mathbb{R}^n}\frac{|u(x)-u(y)|^p}{|x-y|^{2n}}\,dx\,dy)^{1/p}$.

The author also develops a theory of the Besov B_p-capacity on $\mathbb{R}^n$ and proves that this capacity is a Choquet set function. Moreover, the author obtains lower estimates of the Besov B_p-capacities in terms of the Hausdorff content associated with gauge functions h satisfying the condition $\int_0^1 h(t)^{p'-1}\frac{dt}{t} < \infty$, where $1/p + 1/p' = 1$.

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MSC:

46E35 Sobolev spaces and other spaces of "smooth" functions, embedding theorems, trace theorems
31C99 Generalizations of potential theory
30C99 Geometric function theory

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