Kobayashi, Tsuyoshi; **Qiu, Ruifeng**

The amalgamation of high distance Heegaard splittings is always efficient. (English)

Zbl 1140.57012
Math. Ann. 341, No. 3, 707-715 (2008).

Summary: Let M be a compact orientable 3-manifold, and F be an essential closed surface which cuts M into two 3-manifolds M_1 and M_2. Let $M_i = V_i \cup_{S_i} W_i$ be a Heegaard splitting for $i = 1, 2$. We denote by $d(S_i)$ the distance of $V_i \cup_{S_i} W_i$. If $d(S_1), d(S_2) \geq 2(g(M_1) + g(M_2) - g(F))$, then M has a unique minimal Heegaard splitting up to isotopy, i.e. the amalgamation of $V_1 \cup_{S_1} W_1$ and $V_2 \cup_{S_2} W_2$.

MSC:

57N10 Topology of general 3-manifolds (MSC2010)
57M25 Knots and links in the 3-sphere (MSC2010)

Full Text: DOI

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