Hwang, Jun-Muk; **Nagai, Yasunari**

Algebraic complete integrability of an integrable system of Beauville. (English)

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The authors consider the following integrable system constructed by Beauville. Given a $K3$ surface S of degree 6 in $\mathbb{P}^4$, let us consider the moduli space M_S^v of stable bundles with rank 2, $c_1 = 0$ and $c_2 = 4$ over S. This space in endowed by the Mukai symplectic structure. Let a smooth cubic threefold $X \subset \mathbb{P}^4$ contains S and M_X^v be the moduli space of stable bundles with rank 2, $c_1 = 0$ and $c_2 = 2$ over X. The restriction of bundles over X to S is an embedding $M_X^v \to M_S^v$ whose fibers are Lagrangian submanifolds. Varying X we obtain a fibration of an open dense subset of M_S^v by Lagrangian submanifolds which are invariant sets of some integrable Hamiltonian system, the Beauville system.

The authors prove the following theorem: Let $\bar{M}_S$ be the moduli space of semi-stable sheaves which compatifies M_S^v and $\tilde{M}_S$ be the O'Grady's resolution of M_S. Then there exists an open subset $U \subset \tilde{M}_S$ such that the Beauville system extends to U and any fiber is the complement of a subvariety of codimension 2 in the intermediate Jacobian $J(X)$. In particular, this implies that for any threefold X the Hamiltonian vector fields on M_X^v are extended to $J(X)$, i.e. the system is algebraically completely integrable.

Reviewer: Iskander A. Taimanov (Novosibirsk)

MSC:

14J60 Vector bundles on surfaces and higher-dimensional varieties, and their moduli
37J35 Completely integrable finite-dimensional Hamiltonian systems, integration methods, integrability tests

Keywords:

moduli space of stable sheaves; $K3$ surface; Lagrangian fibration; integrable system

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References:

[1] Beauville, Arnaud, Symposium in Honor of C. H. Clemens (Salt Lake City, UT, 2000), 312, Vector bundles on the cubic threefold, 71-86, (2002), Amer. Math. Soc., Providence, RI · Zbl 1056.14059

[2] Druel, Stéphane, Espace des modules de faisceaux de rang 2 semi-stables de classes de Chern \(c_1=0, c_2=2\) et \(c_3=0\) sur la cubique de \(\mathbb{P}^4,\) Internat. Math. Res. Notices, 19, 985-1004, (2000) · Zbl 1024.14004

[3] Huybrechts, Daniel; Lehn, Manfred, The Geometry of Moduli Spaces of Sheaves, (1997), Friedr. Vieweg \& Sohn, Braunschweig · Zbl 0872.14002

[4] Kirwan, Frances Clare, Partial desingularisations of quotients of nonsingular varieties and their Betti numbers, Ann. of Math. (2), 122, 1, 41-85, (1985) · Zbl 0592.14011

[5] Lehn, Manfred; Sorger, Christoph, La singularité de O'grady, J. Alg. Geom., 15, 4, 753-770, (2006) · Zbl 1156.14030

[6] Mukai, Shigeru, Symplectic structure of the moduli space of sheaves on an abelian or \(K3\) surface, Invent. Math., 77, 1, 101-116, (1984) · Zbl 0565.14002

[7] O'Grady, Kieran G., Desingularized moduli spaces of sheaves on a \(K3,\) J. Reine Angew. Math., 512, 49-117, (1999) · Zbl 0928.14029