Broutin, N.; **Devroye, L.**; **McLeish, E.**; **de la Salle, M.**
The height of increasing trees. (English) Zbl 1148.05024
Random Struct. Algorithms 32, No. 4, 494-518 (2008).

Summary: We extend results about heights of random trees [*L. Devroye*, J. Assoc. Comput. Mach. 33, No. 3, 489–498 (1986; Zbl 0741.05062), SIAM J. Comp. 28, No. 2, 409–432 (1998; Zbl 0915.68089)]. In this paper, a general split tree model is considered in which the normalized subtree sizes of nodes converge in distribution. The height of these trees is shown to be in probability asymptotic to $c \log n$ for some constant c. We apply our results to obtain a law of large numbers for the height of all polynomial varieties of increasing trees [*F. Bergeron*, *P. Flajolet* and *B. Salvy*, Lect. Not. Comp. Sci. 581, 24–48 (1992)].

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05C05 Trees
05C80 Random graphs (graph-theoretic aspects)

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