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Weighted height of random trees. (English) Zbl 1147.68058

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Summary: We consider a model of random trees similar to the split trees of *L. Devroye* [*SIAM J. Comput.* 28, 409–432 (1998; [Zbl 0915.68089](#))] in which a set of items is recursively partitioned. Our model allows for more flexibility in the choice of the partitioning procedure, and has weighted edges. We prove that for this model, the height H_n of a random tree is asymptotic to $c \log n$ in probability for a constant c that is uniquely characterized in terms of multivariate large deviations rate functions. This extension permits us to obtain the height of pebbled tries, pebbled ternary search tries, d -ary pyramids, and to study geometric properties of partitions generated by k - d trees. The model also includes all polynomial families of increasing trees recently studied by *N. Broutin, L. Devroye, E. McLeish* and *M. de la Salle* [“The height of increasing trees”, *Random Struct. Algorithms* 32, 494–518 (2008; [Zbl 1148.05024](#))].

MSC:

68R10 Graph theory (including graph drawing) in computer science
05C05 Trees
05C80 Random graphs (graph-theoretic aspects)
68P05 Data structures

Cited in **3** Documents

Software:

[PATRICIA](#); [Quicksort](#)

Full Text: [DOI](#)

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